JETS

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Some History and Introduction

At the time of the previous conference of this series (Tbilisi, 1976) data were presented by Hansen et al. 1) (SLAC) that constituted the first indication for the existence of "quark" jets. Hadrons produced in $e^+e^-$ annihilation at energies ranging from $Q=3\text{GeV}$ to $Q=7\text{GeV}$ were analyzed in two extreme models. One of them was a "phase space" model wherein hadrons were assumed to be produced in random directions. This model fits the lower energy data but disagrees significantly with the higher energy data. The second model was a "two step" (quark production plus soft hadronization) model that consistently described all the data.

In the first step the virtual photon is assumed to materialize into two pointlike spin $\frac{1}{2}$ quarks. This implies a distribution of outgoing energy $\sim 1 + \cos^2 \theta$, with $\theta$ the angle between the lepton beam and the quarks. In a second step or longer time scale, the outgoing quarks were assumed to materialize into hadrons with an exponentially damped transverse momentum ($\exp(-4p_T^2)$ or $\exp(-6p_T^2)$) relative to the quark axis: the would-be jet axis. This model was successful in describing the data in all the available energy range and, moreover, the measured jet axis distribution (found by somehow minimizing event per event, the transverse momentum) agreed with the expected $1+\cos^2 \theta$. This is evidence for the spin $\frac{1}{2}$ nature of quarks (scalar quarks, for instance, would give a distribution $\sim \sin^2 \theta$).

To the eyes of an optimist, this is evidence for QCD, the only realistic asymptotically free theory we know where it makes sense to draw a quark-production diagram as the leading contribution in perturbation theory. We may also conclude that whatever incompletely understood process turns quarks into hadrons, it is soft enough not to obliterate at sufficiently high energy the quark (jet) axis information, while it is certainly enough to erase the memory of the assumed quark quantum numbers (color and fractional charge).

QCD perturbation theory for $e^+e^-$ annihilation stands on a relatively firm basis. The corresponding statement is not true for models of the hadronization process that turns the "primordial" QCD quanta into real hadrons. Purists have advocated a method to circumvent the non-perturbative problem of hadronization, rather than solve it. The idea is to "average" the information on the way that hadronic energy flows out of an interaction, over angular widths larger than the natural uncertainty introduced by the (assumed) limited $\langle p_T^2 \rangle$ non-perturbative process that turns quarks and gluons into real particles. This allegedly erases the reference to the details of long-distance non-perturbative physics. In practice it amounts to measuring jet cross-sections rather than individual
hadron cross-sections. The overall jet directions and momenta are then interpreted as the corresponding quantities for the QCD quanta that allegedly begat the jets. The cross sections for hard and non-collinear QCD quanta, hopefully observable as jets, can be consistently computed in perturbation theory. Thus, the hope is that jet cross-sections will be tests of perturbative QCD. The analysis of quark jets described in the first paragraph was an example of such a procedure successfully carried out for the leading perturbative QCD prediction (no hard non-collinear gluons emitted) in $e^+e^-$ annihilation.

For the actual analysis of experiments and to develop guidelines for the construction of future detectors, it may be useful to discuss models of hadronization, rather than to attempt, as in the previous paragraph, to sweep the problem under the rug. Most QCD-inspired models of hadronization are string-like models. The primordial quarks produced in $e^+e^-$ annihilation lose momentum as they separate in space in favor of energy density in a color flux tube extending from one quark to the other. In confining theories, the flux tube should have transverse dimensions of a typical inverse-few-hundred-MeV size. The energy density in this colored tube materializes in the form of quark pairs of few-hundred MeV momentum in the $q\bar{q}$ c.m.s. The process repeats itself till $(q\bar{q})$ objects of mass $m_\rho$ or $m_\eta$ are reached. Simplistic models of this type predict logarithmically rising multiplicities, flat rapidity distributions at very high energies and a strong suppression of heavy secondary-quark production. I will return to these models when discussing gluon jets.

So much for what was known two years ago. I now turn to the discussion of present and future problems:

1. What are quark jets going to look like at higher than explored energies?
2. Does QCD work beyond the trivial order of $q\bar{q}$ production?
3. Are there $\text{Gl}_{\text{uon}}$ jets?
4. Do gluons have spin 1?
5. What are the properties of gluon jets?

For simplicity, and due to the time constraints, I discuss these problems in the simple context of $e^+e^-$ annihilation.

**High Energy Properties of Quark Jets, Infrared-Safe Variables and $\Upsilon$ Decays**

The "low energy quark jets" (Q=3 to 7 GeV) observed at SLAC and DESY\(^3\) (Q=3.1 to 9.5 GeV) become narrower with increasing energy at a very fast pace.
The data in this energy region are compatible with a logarithmically rising multiplicity $N$ and a fixed $<p_T> \sim 400$ MeV per particle. The angular spread $\delta$ of an average jet decreases as $\delta \propto <p_T>/Q \propto \log Q/Q$. In renormalizable field theoretic models like QCD, however, dimensionless quantities, like $\delta$, scale (up to logarithms). The perturbative QCD prediction for the angular spread $\delta$ of a quark – antiquark plus gluon final state is $\delta \propto O(\alpha_s)$ or $\propto \exp(-1/\alpha_s)$, with $\alpha_s$ the renormalization group improved running coupling constant $\propto (\log Q)^{-1}$. Thus, for large enough $Q$, the chromodynamic prediction for the "width" of a jet is larger than the prediction of a fixed $p_T$ model. The properties of jets in a QCD context have been studied by various authors in complementary ways.

Sterman and Weinberg$^4$ computed to first non-trivial order in QCD ($O(\alpha_s)$) the fraction $f$ of events that would have all but a fraction $\varepsilon$ of the total available energy contained within two opposite cones of half angle $\delta$. The necessity to use a sophisticated observable like the one at hand stems from the need to avoid the infrared sensitivity of perturbation theory. The result is$^5$

$$ f = 1 - \frac{4}{3} \frac{\alpha_s(Q^2)}{\pi} \left[ \ln \delta \left( 4 \ln \delta + 3 + 8 \varepsilon^2 - 4 \varepsilon \right) + \frac{\pi^2}{3} - \frac{5}{2} \right] $$

The limits of validity of this formula are rather narrow, $\varepsilon < 0.1$ to be away from the zero in the coefficient of $\log \delta$ and $\delta > 20^\circ$ for perturbation theory to make sense at, say $Q=10$GeV, $\varepsilon = 0.1$. The corresponding formula for gluon jets has been derived by several authors$^6$. Both formulas have been "renormalization group improved" by several authors$^7$ in the sense of summing all powers of $(\alpha_s \log \delta)^n$ that occur to higher orders of perturbation theory. These perturbative QCD formulas for the width of energy-flow distributions (analyzed with a two-jet bias) would be considered pessimistically or even catastrophically large by some jet-detector-builders. Perhaps this pessimism is not fully justified. The large predictions for the angular width of jets $\delta$ originate from the likelihood of hard non-collinear gluon bremsstrahlung. At sufficiently large energy it is quite possible that the "large" jets do show up as two separable "narrow" jets or, at least, as very oblate jets. More on this later.

Georgi and Machacek$^8$ and Fahri$^9$ have investigated the $O(\alpha_s)$ perturbative-QCD properties of jets in terms of averages of variables (sphericity and thrust, respectively) designed to give a simple, rough and infrared-safe idea of how jetty events are. Their results have been extended to the decays of "onium" resonances and to differential distributions by the authors of Refs. 2 and 10).
Spherocity is defined as
\[ S = \left\{ \frac{4}{\pi} \min \frac{\sum_i |p_T^i|}{\sum_i |p_i|} \right\}^2 \]
The sums run over all hadrons in the final state. The \( p_T^i \) are momenta transverse to an axis that is determined on an event-by-event basis by minimizing \( S \). The value \( S=0 \) corresponds to a perfectly collimated jet and \( S=1 \) to a spherical distribution of momentum in the final state. At sufficiently large energies a preliminary analysis supports the hope that it does not make a large practical difference to include or not to include neutrals in the sum defining \( S \).

Thrust is somewhat complementary to \( S \) and is defined as:
\[ T = \max \frac{\sum_i |p_{\parallel}^i|}{\sum_i |p_i|} \]
where \( p_{\parallel}^i \) are momenta parallel to an axis chosen to maximize \( T \). Thrust reaches unity for a perfect jet and \( T=\frac{1}{2} \) for a spherical momentum distribution. The thrust of the arguments can be summarized in a picture, Fig.1. The average value \( 1-\langle T(Q^2) \rangle \) is shown as a function of \( Q \) in GeV. The curve labelled \( NP \) is an estimate of the "non-perturbative" effects assumed to give rise to a fixed \( \langle p_T \rangle \) distribution around the jet axis. The curve labelled QCD is a first order perturbative calculation for e+e- + qq, q̅ + gluon. At about 10 GeV perturbative hard gluon emission becomes more important than the NP effect (part of which, at low energy, no doubt is the "QCD" contribution). The \( T \) is predicted to decay via three-gluons: a planar T=1 configuration even to the leading order of QCD. Thus, at \( Q = m_T \), \( 1-T \) is predicted to jump dramatically. The expectation is qualitatively and quantitatively born by the data. The behaviour of \( \langle S(Q^2) \rangle \) is very similar to Fig.1, the quantities \( S \) and \( 1-T \) being highly correlated.

The analysis of the distributions \( d\sigma/dS \) or \( d\sigma/dT \) may prove instrumental in jet-hunts, particularly gluon jet-hunts. Predictions along these lines are given in Fig.2 for \( T \) decays. Fig.2a is the predicted thrust distribution for \( \pi^0 \) decays. The circles in the rest of the figure are polar plots of predictions for the "Pointing" vector in the decay plane (the plane relative to which the transverse momentum is minimal). The Pointing vector is the one-particle
distribution \( p(\phi) d\phi / d\phi \), where \( p \) is the projection of the particle momentum onto the decay plane and \( \phi = 0 \) is the thrust axis. The slice \( 60^\circ < \phi < 180^\circ \) is defined by having more momentum \( \Sigma |p_i| \) in it than the slice \( 180^\circ < \phi < 300^\circ \). The l.h.s. of the figure contains perturbative QCD predictions for the Pointing vector. As the thrust (corresponding to the mightiest gluon) diminishes, the two others are forced by 3-body decay kinematics to point in increasingly separated directions. At \( T = e^2/3 \), (the minimum value for a planar decay) the Mercedes-Benz limit is reached but the leading order QCD cross-section vanishes. Some of the QCD three-jet structure may survive the non-perturbative barrier of effects for a statistically significant ensemble of events. The r.h.s. of the picture is an estimate of what the Pointing vector may actually look like in a model where relatively low energy gluon jets have a somewhat larger multiplicity but not significantly tougher \( \langle p_T \rangle \) than quark jets \(^2\).

Do Gluons Have Spin 1?  

The \( i \cos^2 \theta \) distribution of the jet axis in \( e^+ e^- \) annihilation is one of the clearest evidences for the existence of spin 1/2 quarks. It would be quite crucial for QCD to have equally simple and feasible tests of the existence of fundamental spin 1 fields. Thrust and sphericity average values and distributions are not very sensitive to the assumed quantum numbers of gluons. Neither is the orientation of the decay plane in low thrust events relative to the \( e^+ e^- \) axis. What follows is an example of a clean test of the spin 1 nature of gluons, one that may unfortunately require a very good \( \gamma \) ray detector.

Suppose one is sitting at an \( e^+ e^- \) machine at \( Q = \) the mass of a radially excited "onium", the \( T' \), say. Some of the time the \( T' \) will \( \gamma \)-ray decay into a P-wave state, the \( J^{PC} = 2^{++} \) one, say. According to the QCD perturbation theory rules, the \( 2^{++} \) decays mainly into two gluons, of \( \sim 5 \text{ GeV} \) momentum each in the example at hand. Such hard gluons are hopefully jetty enough for the determination of the \( 2^{++} \) decay jet axis to be feasible. Thus one may compute angular distributions for this sequential decay in terms of the angles between the \( e^+ e^- \) axis, the jet axis and the \( \gamma \) ray direction. Let these be \( \theta_{e\gamma}, \theta_{ej} \) and \( \theta_{\gamma j} \) in an obvious notation. Predictions for distributions in these angles are given in Fig. 3, for the example at hand\(^2\). The top figure is for the orthodox decay into vector gluons. The middle figure is for a decay into hypothetical spin 0 gluons or a "direct" decay into a light \( qq \) pair with zero helicity. The bottom figure is for light quarks with helicity one. Notice how different the orthodox prediction is from the alternative
examples. Not too many data may be needed to thus "measure" the gluon spin.

Properties of Gluon Jets

The analysis of gluon jet widths in first order QCD perturbation theory \(^6\) and to all orders in the leading log approximation \(^7\) results in the somewhat pessimistic conclusion that gluon jets will be much wider than quark jets. This difference partially originates in the fact that a color-octet charge (gluons) is a stronger charge than a color-triplet charge (quarks). Thus gluons radiate more gluons than quarks do. If this expectation is borne by experiment, gluon jets may not be obvious to the naked eye unlike quark jets at, say, Q=10GeV. This is not as sad as it sounds at first: analysis of thrust and sphericity distributions may still be fine checks of QCD, the analysis of statistical ensembles of events in terms of antenna patterns (the Pointing vector) may still produce results that are pleasantly jet-like \(^2\).

Decays ofonium resonances and low thrust events in the continuum provide an exhilarating testing ground for models of hadronization: the incompletely understood "long distance" process whereby quarks and gluons turn into real particles, mainly pions. What follows in an extreme model \(^14\), intended only as an example, perhaps to be soon discredited by experiment. Suppose the colored string model of quark hadronization described in the introduction is roughly right. But suppose one is concerned with high sphericity or low thrust events for which one of several hard gluons are allegedly and ultimately responsible. The question is how three hard and non-collinear quanta (two quarks and a gluon in the continuum or three gluons in an \(T\) decay) turn into hadrons. Suppose the properties of the strings of color flux are the ones that one may infer from lattice gauge theories: this is the extremism of the model to which I have referred. In this context, a gluon string (the flux lines coming out of an octet source) has more than twice the tension and the energy per unit length of a quark string. Also, a gluon string may split into two quark strings. With these considerations in mind, one is tempted to conclude that the most energetically favourable configuration for the development of strings in \(e^+e^-q\bar{q}\) is the one in Fig.4e, while the most favourable configuration in \(\gamma^*ggg\) decays is the one in Fig.4f. The next step is to guess that, at least in an average sense, the \(e^+e^-q\bar{q}\) decay will be like the decay of a couple of stringed quarks at \(Q^2 = (k_1 + k_3/2)^2\) plus another couple of stringed quarks with \(Q^2 = (k_2 + k_3/2)^2\). It suffices then to take the bulk of the \(e^+e^-\) data at
these lower masses, boost it to the relevant moving frame, weigh it with the chromodynamic probability for $e^+ e^- qar{q}$, and there comes your answer. The same considerations apply to $\gamma^* g g$ decays, all of whose details can in this extreme model be predicted from $e^+ e^-$ data in the continuum. This analysis produces predictions for Pointing vectors that are not unlike the ones of Fig. 2, except that they are farther away from the prominent peaks. Multiplicities for $T$ decay are larger than in the continuum but are in fact related to $e^+ e^- q\bar{q}$ multiplicities at a much lower momentum scale. Details of these considerations appear in Ref.14.
REFERENCES


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7) R. Patronzio - CERN Preprint TH. 2571 (1978);


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13) The naïve prediction is the same as in orthopositronium decay; A. Ore and J.L. Powell - Phys. Rev. 75 (1949) 1696.

Figure 1: Graph showing the behavior of $\langle 1-T \rangle$ as a function of $Q$ (GeV). The graph includes curves labeled QCD, QCD+heavy quark decay, and NP, with peaks at $\gamma$, $\gamma'$, and $\gamma''$. The x-axis represents $Q$ (GeV) ranging from 3 to 500, while the y-axis represents $\langle 1-T \rangle$ ranging from 0.15 to 0.05.
- Figure 2 -
Jet angular distributions: $2\gamma$ decays

- Figure 3 -