FACTORORIZATION BREAKING IN DEEP INELASTIC NEUTRINO
HADRON production IN PERTURBATIVE QCD

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ABSTRACT

In the naive parton model the single hadron inclusive cross-section of a deep inelastic neutrino reaction factorizes $W(X,Z) = f(X)d(Z)$. The next-to-leading order correction in perturbative QCD is calculated and gives the breakdown of factorization. We propose to test the prediction by taking ratios of double moments of structure functions integrated over produced hadron $P_T$.

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Recent high-energy neutrino experiments have provided a nice testing ground for perturbative QCD \(^1\). The naive parton model predicts that the structure function \(W(X,Z)\) of hadron production is given by a product of the quark distribution function \(f(x)\) in the target and the quark fragmentation function \(D(Z)\) into hadrons \(^2\). Perturbative QCD predicts factorization in leading order and provides corrections to it in powers of the running coupling constant \(\frac{\alpha}{\pi}\) as well as the \(Q^2\) dependence to parton distribution and fragmentation functions. The purpose of this letter is to present the next-to-leading order calculation in perturbative QCD and to propose a test of the predicted factorization breaking for single hadron inclusive production in neutrino reactions.

Without measuring transverse momenta of hadrons, we obtain only three invariant structure functions for \(\nu + \text{target} (P_1) \to \bar{\nu} + \text{hadron} (P_2) + \text{anything}\)

\[
W_{\mu\nu}^{\nu} = d^{\mu\nu}W_2 + \bar{g}^{\mu\nu}W_3 + i\epsilon^{\mu\nu\rho\sigma}\frac{P_2\alpha}{P_1\cdot q}W_3
\]

\[
d^{\mu\nu} = -\bar{g}^{\mu\nu} + \bar{g}^{\mu\sigma}P_1^\sigma - \bar{g}^{\nu\sigma}P_1^\sigma / P_1^\alpha \bar{g}_{\alpha\beta}P_1^\beta
\]

where \(\bar{g}^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu / q^2\), \(q^\mu\) being the four-momentum of the weak current, and \(W_i\) are functions of \(X = -q^2 / 2P_1q\), \(Z = 2P_2q / q^2\) and \(q^2 = -Q^2\). We take \(Z > 0\) to avoid target fragmentation \(^3\).

In the naive parton model, structure functions of pion production from \(\nu\) nucleon collisions are given in terms of the \(a\) th quark distribution function \(f^a\) and the \(b\) th quark fragmentation function \(D^a_b\) for instance \(^*\)

\[
W_2^{\nu\to\pi} = (f_d + f_{\bar{u}})(D_u^\pi \cos^2 \theta + D_c^\pi \sin^2 \theta) + f^S D_c^\pi
\]

\(^*\) Another possible longitudinal variable is \(P_1P_2 / P_1q\). In this case one needs to look into large \(P_{2T}\) or azimuthal angular distributions to avoid target fragments.

\(^*\) We neglect sea quark contributions multiplied by \(\sin^2 \theta\) (Cabibbo angle).
Since pions are produced from a charmed quark only through gluons, we can eliminate $D_C$ by taking the difference of $\bar{u}$ and $\bar{d}$, and obtain a factorized form:

$$W_2^{\nu\to\pi^+} - W_2^{\nu\to\pi^-} = (f^d + f^{\bar{u}})(D_\nu^\pi + D_\nu^\pi)\cos^2\theta$$

(4)

Another way of obtaining a factorized expression is to take the difference between proton and neutron target data (non-singlet in target channel)

$$W_2^{\nu p\to\pi^+} - W_2^{\nu n\to\pi^-} = (f^d - f^d - f^{\bar{u}} - f^{\bar{u}})(D_\nu^\pi \cos^2\theta + D_c^\pi \sin^2\theta)$$

(5)

Even if we have primordial $p_T$ associated with quark distribution and fragmentation functions, by using a covariant parton model approach we can show that the factorization prediction of the naive parton model is unaffected provided transverse momenta of produced hadrons are integrated: $W_i^{\nu}(x, q^2) = W_i^{\nu}(x, R(2))$, where $W_i^{\nu}$ are usual structure functions of fully inclusive deep inelastic scattering.

By using a parton model interpretation of the operator-product expansion we have also found that target mass corrections can be incorporated by the $\xi$ scaling formula.

It has been shown that mass singularities of inclusive cross-sections with partons as beam, target, or produced particles factor to all orders in perturbation theory of QCD: parton structure function $W_i^{ab}$ for the neutrino reaction $\nu a \to bX$, where $a, b = \text{quark, antiquark, or gluon}$, is given by taking moments

$$W_i^{ab, nm} \equiv \int_0^1 dx x^{-i} \int_0^1 dy y^{-m} W_i^{ab}(x, y)$$

$$= A_n^a \left( \frac{Q^2}{p_i^2} \right) \Gamma_m^b \left( \frac{Q^2}{p_2^2} \right) C_i^{ab, nm}$$

(6)

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\* If one identifies charge, but not particle species reliably (e.g., in many of the bubble chamber experiments), one may still have a small contamination from $D_C^\pi - D_C^\pi 
eq 0$.

\# We neglect masses of quarks and produced hadrons.

\#\# We use small letters for partons and capitals for hadrons.
where $A^a_n$ is the quark operator matrix element between the $a^{th}$ parton states and $\Gamma^b_m$ is called the (time-like) cut vertex. Following the perturbative QCD recipe we assume that mass singularities in $A^a_n$ and $\Gamma^b_m$ can be cancelled by multiplying the factors $\bar{f}^a_n$ and $D^b_m$

\[ f^a_n(Q^2) = \bar{f}^a_n A^a_n(Q^2/p_i^2) \]  

\[ D_{b,m}(Q^2) = \bar{D}^b_m \Gamma^b_m(Q^2/p_2^2) \]  

(7)  

(8)  

to give the $Q^2$ dependent parton distribution function $f^a(x,Q^2)$ and the fragmentation function $D_b(2,Q^2)$. The hadronic structure function is then given by

\[ W_{i,nm} = \sum_{a,b} f^a_n(Q^2) D_{b,m}(Q^2) C_{i,nm}^{a,b} \]  

(9)  

In leading order we obtain that

\[ C_{2,nm}^{qq} = C_{2,nm}^{\bar{q}\bar{q}} = C_{3,nm}^{qq} = C_{3,nm}^{\bar{q}\bar{q}} = 1 \]  

and all others vanish. We can calculate the "coefficient function" $C_{1,nm}^{ab}$ perturbatively, but to obtain $f^a_n(Q^2)$ and $D^b_m(Q^2)$ we have to fit data at some $Q^2$.

To do an unambiguous test of perturbative QCD prediction, we propose to study if the structure function $W(X,Z)$ deviates from a product of functions of $X$ and $Z$. Namely, one should examine deviations from unity of the following ratio of double moments which does not involve $f_n$ and $D_m$ **

\[ \frac{W_{nm} W_{kl}}{W_{km} W_{nl}} = \frac{C_{nm} C_{kl}}{C_{nl} C_{km}} = 1 + O(Q^2) \]  

(10)  

*) We neglect effects of primordial $p_T$ and target mass in calculating the next-to-leading order correction.

**) We suppress the suffix $i = 2, 3$, temporarily. We consider the non-singlet case where only $W_q + qX$ (or $q$) contributes. The mixing problem is discussed at the end.
To obtain the coefficient function $C_{nm}$ we use Eq. (6) and subtract contributions of operator matrix elements and cut vertices from the parton structure function $w_{nm}$. The resulting coefficient function has the general form:

$$C_{nm} = 1 + \frac{g^2}{16\pi^2} \left( a + b_n + c_m + d_{nm} \right)$$

$$\sim \left( 1 + \frac{g^2}{16\pi^2} \left( \frac{a}{2} + b_n \right) \right) \left( 1 + \frac{g^2}{16\pi^2} \left( \frac{a}{2} + c_m \right) \right) \left( 1 + \frac{g^2}{16\pi^2} d_{nm} \right)$$

where $\frac{g^2}{16\pi^2} = (B_0 \ln Q^2/\Lambda^2)^{-1}$ with $B_0 = 11 - 2N_c/3$. Equations (6) and (11) show that operator matrix elements and cut vertices affect only $a$, $b_n$, and $c_m$, respectively. Therefore, $d_{nm}$ and $b_n$ can be obtained without calculating cut vertices $\Gamma_m$. The ratio (10) is then given by

$$\frac{w_{nm} w_{k\ell}}{w_{km} w_{n\ell}} = 1 + \frac{g^2}{16\pi^2} d_{nm, k\ell}$$

$$d_{nm, k\ell} = d_{nm} - d_{km} + d_{k\ell} - d_{n\ell}$$

As a second test we can examine if the parton distribution function in Eq. (9) is the same one that appeared in the fully inclusive deep inelastic structure function $W^t(x)$ by studying the ratio $w_{nm} w_{k\ell}^t / w_{km}^t w_{n\ell}^t$. Using the next-to-leading correction $B_n$ to the coefficient function for $w_n^t$ in Ref. 7), we obtain

$$\frac{w_{nm} w_{k\ell}^t}{w_{km}^t w_{n\ell}} = 1 + \frac{g^2}{16\pi^2} \left( b_n + d_{nm} - B_n - b_m - d_{km} + B_k \right)$$

Since the cut vertex is common to all three structure functions $W_i$, we can perform the above tests (12) and (14) by taking ratios between any (possibly different) linear combinations of $W_i$. In particular, the cross-section itself $d\sigma/dx dy dz$ or its integral over $y$ can be used instead of $W_i$.

* For definiteness we define $a$, $b$, and $c$ to correspond to terms in $G(x, z)$ with $\delta(1-x)\delta(1-z)$, $\delta(1-x)$, and $\delta(1-z)$, respectively, and $d(x, z)$ to be free of $\delta(1-x)$ and $\delta(1-z)$.

** Consequently, the dependence on renormalization schemes and gluon gauges sits only in $a$, $b_n$, and $c_m$.

*** The prediction for $w_n^t$ can best be tested separately without taking ratios, because it vanishes in leading order and has no renormalization ambiguities to order $g^2$. 
If one wants to study the next-to-leading correction completely, one needs to calculate the anomalous dimension of order $\tilde{g}^n$. Existing calculations for operator matrix elements $A_n$ and for cut vertices $\Gamma_m$ employed different renormalization schemes and should not be combined together. Here we choose the minimal subtraction scheme of Refs 7) and 8) and postpone the calculation of cut vertices to a later paper.

We calculate perturbative QCD corrections by putting partons infinitesimally off-shell to regularize infra-red and mass singularities. Double logarithms due to infra-red singularities are cancelled by virtual gluon corrections. We find that terms $a_n, b_n, c_m$ are common to $W_2$ and $W_3$ whereas $W_5$ has only $d_{nm}$. We present $d_{nm}$ and $b_n$ for all combinations of quark, antiquark, and for all three structure functions $W_i$.

1) $W_q \to qX$ (see Fig. 1)

\begin{align}
   d_{3nm}^{qq} &= \frac{8}{3} \left( \sum_{i=1}^{n-1} \frac{1}{i} \sum_{j=1}^{m-1} \frac{1}{j} \right) + \frac{3}{4} \sum_{j=1}^{n+1} \left( \sum_{i=1}^{m+1} \frac{1}{i} \right) + \frac{2}{(n+2)(m+1)} \\
   d_{2nm}^{qq} &= d_{3nm}^{qq} + \frac{16}{3} \left\{ \frac{3}{(n+2)m} - \frac{3}{(n+3)(m+1)} - \frac{1}{(n+2)(m+1)} \right\} \\
   d_{bnm}^{qq} &= \frac{32}{3m} \left\{ \frac{1}{n+2} - \frac{1}{(n+3)(m+1)} \right\} \\
   b_n^{qq} &= \frac{8}{3} \left\{ \left( \sum_{j=1}^{n} \frac{1}{j} \right)^2 \sum_{j=1}^{m-1} \frac{1}{j^2} - \frac{1}{n(n+1)} \sum_{j=2}^{n} \frac{1}{j} + \frac{2\pi^2}{3} - \frac{7}{2} \right\}
\end{align}

2) $W_q \to gX$, $m \geq 2$ (see Fig. 2)

\begin{align}
   d_{3nm}^{qG} &= \frac{8}{3} \left\{ \frac{1}{(n+2)m} \left( \frac{1}{m+1} - \frac{1}{n+1} \right) + \sum_{j=1}^{n+2} \frac{1}{j} \right\} - (m-1)! \sum_{i=1}^{\infty} \frac{(i-1)!}{(m+i-1)!} \left( \sum_{j=1}^{m+i-1} \frac{1}{j} \right) + \sum_{j=1}^{m+i+1} \frac{1}{j} \right) \\
   d_{2nm}^{qG} &= d_{3nm}^{qG} + \frac{16}{3m} \left\{ \frac{3}{(n+3)(m+1)} + \frac{1}{(n+1)(m+2)} - \frac{1}{(n+2)(m+1)} \right\} \\
   d_{8nm}^{qG} &= \frac{4}{3} \frac{8}{(n+3)(m+1)}
\end{align}

whereas $b_n^{qG} = 0$. 
3) $WG \to qX$ $^a)$ (see Fig. 3)

\[ d_{3\,nm}^{Gq} = - \frac{n^2 + n + 2}{n(n+1)(n+2)} \sum_{j=1}^{m-1} \frac{1}{j} \]

\[ - (m-1)! \sum_{i=1}^{\infty} \frac{(n+i)^2 + (n+i) + 2}{(n+i)(n+i+1)(n+i+2)(m+i-1)!} \]

\[ d_{2\,nm}^{Gq} = - d_{3\,nm}^{Gq} - \frac{2(n^2 + n + 2)}{n(n+1)(n+2)} \sum_{j=1}^{m-1} \frac{1}{j} + \frac{2}{m} \left( \frac{6}{(n+2)(n+3)} - \frac{1}{n+1} \right) \]

\[ d_{s\,nm}^{Gq} = \frac{8}{(n+2)(n+3)m} \]

\[ b_{n}^{Gq} = \frac{1}{n^2} - \frac{n^2 + n + 2}{n(n+1)(n+2)} \sum_{j=1}^{n} \frac{1}{j} \]

If $q$ is replaced by $\bar{q}$, $C_3 \to -C_3$ (relative sign of Born and next-to-leading terms stays the same).

The cleanest test may be done by taking a non-singlet in both target and produced hadron channels (i.e., by taking the differences of charge states). For this non-singlet combination of $W_3$ we obtain a particularly simple prediction for the factorization breaking Eq. (12) with

\[ d_{3\,nm,kl}^{Gq} = \frac{8}{3} \left\{ \sum_{j=1}^{n-1} \frac{1}{j} \sum_{i=1}^{m-1} \frac{1}{i} + \sum_{j=k+2}^{n+1} \frac{1}{j} \sum_{i=k+2}^{m+1} \frac{1}{i} + \frac{2(n-k)(m-l)}{(n+2)(k+2)(m+1)(l+1)} \right\} \]

for $n > k$, $m > l$. Numerically, if $\bar{g}^2/4\pi = 0.5$, the predicted factorization breaking is of the order of 10% or 20% for small $n$ or $m$, but becomes larger as $n$ or $m$ increases: for instance, $W_{22}W_{11}/W_{21}W_{12} = 1.12$ and $W_{33}W_{11}/W_{31}W_{12} = 1.18$ for the non-singlet $W_3$.

$^a)$ It should be noted that $W_1$ has mixing in the target channel in contrast to the fully inclusive structure function $W_3^1$. For the operator matrix element we should take only clockwise quark loops with single flavour because we distinguish quarks and antiquarks here.
Even when the non-singlet part is not separated we can analyze the mixing problem in the following way. Let us take, for instance, $\nu p + \mu^- X$. Perturbative QCD modifies the naive parton model prediction (4) by adding a mixing contribution $2f^G_n (D_{2,n}^+ - D_{2,n}^-) C_{2,n}g_{2,n}^\mu$ where $C_{2,n}g_{2,n}^\mu$ is the coefficient function of $WG + qX$. The ratio $f^G_n/(f^d_n + f^u_n)$ can be obtained by fitting the $Q^2$ dependence due to the mixing in the leading logarithmic approximation. For instance, the ratio of $W_2$ for the difference of $\nu p + \mu^+ X$ is modified from (12) to

$$\frac{(W_{2, nm} - W_{2, nm}^\mu)}{(W_{2, n} - W_{2, n}^\mu)} \left( \frac{W_{2, k}^\mu - W_{2, k}^-}{W_{2, k}^\mu - W_{2, k}^-} \right) = \frac{\bar{g}^2}{16\pi^2} \left\{ d_{2,nm,kl}^q + \frac{2f_n^G (d_{2,nm}^G - d_{2,nl}^G)}{f_n^d + f_n^u} + \frac{2f_k^G (d_{2,k}^q - d_{2,k}^q)}{f_k^d + f_k^u} \right\}$$

(27)

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Figure captions

Fig. 1 : Perturbative QCD diagrams which contribute to $W_q \rightarrow qX$. Wavy,
         solid, and curly lines represent weak currents, quarks, and gluons.

Fig. 2 : Perturbative QCD diagrams for $W_q \rightarrow qX$.

Fig. 3 : Perturbative QCD diagrams for $WG \rightarrow qX$. 
\[ \left( \begin{array}{c} \text{FIG. 1} \\ 2 \end{array} \right) + \left( \begin{array}{c} \text{FIG. 2} \\ 2 \end{array} \right) + \frac{1}{2} \left( \begin{array}{c} \text{FIG. 3} \\ 2 \end{array} \right) \]