PERMION FAMILIES IN SUPERGRAVITY

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The unification of the electromagnetic and weak interactions was proposed a long time ago and achieved after many years with the Weinberg-Salam model, based on the SU(2) x U(1) gauge group. This theory has been consolidated in the last decade. At the same time, the description of the strong interactions by a Yang-Mills theory based on the colour gauge group SU(3) has been corroborated by several phenomenological successes and should eventually lead to a full understanding of hadron physics. Hence the idea of a "grand-unified" theory (GUT)\(^1\) of the strong and electroweak interactions naturally followed, in spite of the dissimilarities among these elementary forces of nature. The gauge group of the three fundamental interactions, SU(3) x SU(2) x U(1) is then embedded in the simple gauge group G of the GUT. The minimal GUT is constructed with G = SU(5), but models have been proposed with larger gauge groups, G \supset SU(5). Besides the conceptual appeal there is a major achievement in this grand-unification approach: the ratios between the coupling constants of the strong, electromagnetic and weak interactions are determined in good agreement with experiment, if G is assumed to be spontaneously broken at an energy around \(10^{13}\) GeV, into the low-energy gauge group SU(3) x SU(2) x U(1). Moreover, the grand unification requires introducing new forces which violate, in particular, the conservation of the baryon number: the GUTs then predict a proton lifetime of about \(10^{31}\) years, a beautiful experimental test of grand unification.

However, several fundamental questions remain unanswered at the GUT level\(^1\). We just enumerate here those which are relevant to our discussion (a very incomplete list):

1) What is the gauge group G? The introduction of a group larger than SU(5) may have several phenomenological motivations but requires the existence of additional particles which should be heavy enough to have escaped observation.

2) What is the representation of G that classifies the elementary fermions? The observed quantum numbers of the leptons and quarks require this representation to be complex in order to naturally explain both the parity violation in weak interactions and the lightness of these fermions. Complex fermion representations usually introduce anomalies (non-renormalizable radiative
corrections) if the gauge group is unitary, \( G = SU(N) \), \( N \geq 5 \). Particular reducible complex representations are then needed in order to have a consistent theory in this case. Since the observed fermions are colour triplets or singlets and doublets or singlets of the weak interaction \( SU(2) \) group, other ("exotic") states are only allowed if they are very massive. In the minimal GUT, with \( G = SU(5) \) these requirements force the fermion representation to be a sequence of \( (5 + 10) \) representations, the so-called fermion families and, at most, additional real representations (corresponding to heavy fermions). For larger groups, the fermion representation has to reduce to the previous one with respect to the \( SU(5) \) subgroup. How many \( (5 + 10) \) families are there in the complex part of the fermion representation? They are at least three, corresponding to the known light leptons and quarks.

3) What are the criteria for choosing the scalar boson multiplets? This choice will select the possible patterns of symmetry breaking and mass generation in the theory.

4) How do we fix the several coupling constants which usually remain free in the \( G \) invariant potential describing the interaction amongst scalars? The values of these parameters will determine the vacuum expectation values of the scalar fields. Moreover, in the minimal scheme, the \( G \) invariance has to be spontaneously broken into \( SU(3) \times SU(2) \times U(1) \) at energies \( 10^{13} \) times larger than the energy where the latter is broken into the \( SU(3) \times U(1)_{\text{em}} \) group. This is the so-called "hierarchy problem", and a ratio of \( O(10^{13}) \) suggests a non-perturbative origin of this hierarchy, requiring a deep understanding of the dynamics.

5) How do we achieve a pattern for the Yukawa couplings of fermions to the scalar bosons which is consistent with the somewhat irregular mass spectrum of leptons and quarks in the three (or more) families?

In this report we will mostly focus our attention on problem 2), namely, the explanation for the repetitive pattern of fermion families.

Global supersymmetry relates particles with different spins\(^2\)). Hence it looks like the right framework to solve many of the open problems in grand unification. But there are several problems in constructing realistic supersymmetric theories\(^3\)). Most of the recent attempts consider just one spinorial charge \( (N = 1 \text{ supersymmetry}) \) and, for example, obtain some insight into problems 4) and 5) but give no explanation for the existence of families of fermions or for the particular choice of scalar fields. Extended supersymmetry (with \( N > 1 \text{ spinorial charges} \)) looks more promising in these respects. For
instance, in the $N = 4$ supersymmetric Yang-Mills theory the fermions are in four adjoint representations of the gauge group $G$. Requiring that each adjoint representation should contain at least one $(\mathbf{5} + \mathbf{10})$ of the SU(5) subgroup, restricts the choice of $G$ to either $G \supset G(11)$ or $G \supset E_6$. The number of $(\mathbf{5} + \mathbf{10})$ fermion families would then be (a multiple of) four. However, $N$ extended supersymmetry has a major problem in that fermions always appear in a real representation of the gauge group. Therefore, in correspondence with the three or more $(\mathbf{5} + \mathbf{10})$ families of light fermions, their SU(5) conjugate states in three or more $(\mathbf{5} + \mathbf{10})$ families are predicted with masses which should all be around the energy where the Weinberg-Salam SU(2) x U(1) gauge symmetry is broken. It is difficult to imagine how one could achieve this mass pattern in a natural way. Anyhow, attempts in this direction would need a better understanding of $N = 4$ super Yang-Mills theories and, in particular, of the spontaneous breaking of the gauge symmetry and global supersymmetry. Up to now, we have discussed the fundamental forces without mentioning gravity. However, at energies of the order of the Planck mass, $M_p$, the gravitational force becomes comparable with any other elementary force and one should wonder what the interplay is between gravity and the GUT. $N$ extended supergravity$^4$ ($4 \leq N \leq 8$) was introduced in physics as an attempt to construct a theory which includes the interaction of the graviton with real matter: gauge bosons, spin 1/2 fermions and scalar bosons. Then $N$ extended supergravity, or local supersymmetry, would provide a framework for superunification, the unification of all elementary forces: strong, electroweak and gravitational interactions. But the quantum dynamical properties of supergravity are still far from being known. Nevertheless, by using its symmetry and supersymmetry properties and by exploiting some similarities with the $\sigma$ model, one can try to foresee how and how much supergravity is compatible with phenomenological requirements, in particular, how would a somewhat visionary insight look in superunification as urged by our faith in grand unification. A fundamental step in this direction has been performed in the work of Cremmer and Julia$^5$: they have shown that $N = 5, 6$ supergravities have a $U(N)$ local symmetry, while $N = 8$ supergravity has a $SU(8)$ local symmetry ($N = 7$ supergravity is believed to be the same as the $N = 8$ one). Interestingly enough, the gauge fields are composite in terms of the elementary fields in the supergravity Lagrangian. Cremmer and Julia suggested that they could be the intermediate bosons of an effective GUT.

Ellis, Gaillard, Maiani and Zumino (EGMZ)$^6$ then made the following conjecture, which is the fundamental hypothesis in the present attempts at superunification. The non-perturbative dynamics of $N$ extended supergravity should be responsible for an effective GUT at energies $< M_p$ with a gauge group.
SU(5) ⊂ G ⊂ SU(N). The fields in the effective GUT, namely gauge bosons, fermions and Higgs scalars, are composite massless states made out of the elementary fields in the N extended supergravity Lagrangian.

These states of the effective GUT have now to be recognized within the massless representations of the N extended supersymmetry\(^7\). Massless supermultiplets which are irreducible representations of N extended supersymmetry are denoted by \(\{\lambda, [k]\}\) where \(\lambda\) is the maximum helicity in the supermultiplet and \(k\) is the SU(N) representation of the corresponding state. We use the notation \([k]\) for the k-fold totally antisymmetric representation of SU(N). The "particle" content of the supermultiplet \(\{\lambda, [k]\}\) is as follows (\(\lambda \geq N/4\), \(k < N\))

\[
\{\lambda, [k]\} = (\lambda, [k]) + (\lambda - 1/2, [N-1] \times [k]) \\
+ \ldots + (\lambda - \ell/2, [N-\ell] \times [k]) + \ldots \\
+ \ldots + (\lambda - N/2, [k]) + PCT conjugate states
\]

For instance, the graviton supermultiplet will be denoted by \(\{2, [0]\}\). Note that the state with helicity \(\lambda_1\) will be in the \([N - 2\lambda + 2\lambda_1] \times [k] + [2\lambda + 2\lambda_1] \times [N - k]\) representation of SU(N).

For further purposes we will now look for supermultiplets containing states with the quantum numbers of the (composite) gauge bosons: \(\lambda = 1\) in the adjoint representation. This implies \(2\lambda + 2 = k\). It is easy to see that for \(N = 1, 2\) there is just one supermultiplet, \(\{3/2, [1]\}\) in our notation, which has such a state. This supermultiplet is usually called "supercurrent multiplet". However, in this supercurrent multiplet, one finds, for \(N \geq 3\), \((N - 2)\) more supermultiplets that contain \(\lambda = 1\) states in the adjoint representation of SU(N). A recent analysis of two-particle composites\(^7,8\) shows that the massless limit of spin one massive currents in the SU(N) adjoint representation indeed has components in several of the supermultiplets discussed above. But since a gauge current is not a Lorentz covariant object this analysis is not conclusive, and it could even happen that for \(N > 2\) the SU(N) gauge currents are indeed in the supercurrent multiplet \(\{3/2, [1]: \frac{N}{2}\}\) as has trivially to be the case for \(N = 1, 2\).

In their attempt at superunification, EGMZ made the following hypothesis: all particles participating in the effective GUT are in the supercurrent multiplet. However, the whole set of spin 1/2 fermions in this supermultiplet is in a complex representation of SU(N) such that the effective GUT would not be anomaly-free. Some criteria are needed to extract an anomaly-free subset of these states.
These criteria have been formulated by Ellis, Gaillard and Zumino (EGZ)\(^9\). Here we will discuss a slightly improved version\(^{10}\) of the EGZ model. Namely we will consider, together with the supercurrent multiplet \([3/2, N]\), the graviton supermultiplet \([2, 1]\), which could be either the elementary ("preonic") supermultiplet of supergravity or a composite one. There are at least three reasons for doing so:

a) one considers supermultiplets containing all the gauge particles in the theory, graviton, gravitini and vector bosons;

b) one avoids the most questionable of the EGZ hypotheses (namely, the exclusion of what EGZ call "trace representations");

c) in their recent study of the Regge trajectories in \(N = 8\) supergravity, Grisaru and Schnitzer\(^{11}\) found that the massless states in the right signature Regge trajectories are precisely in these two multiplets.

The EGZ criteria for extracting an effective GUT are then the following:

I) the gauge group is the larger subgroup of \(SU(N)\) which satisfies hypothesis II), with fermions in a non-real representation.

II) The helicity \(-1/2\) states in the effective GUT are selected as: the maximal (in number of states) set that is vector-like with respect to the colour \(SU(3)\).

III) The remaining fermionic states do not enter the effective GUT so as to save its renormalizability.

With these criteria EGZ found that i) \(N = 8\) supergravity is selected; ii) the effective GUT gauge group is \(SU(5) \times SU(2)\); iii) the theory has exactly three \((5 + 10)\) fermion families together with many other states in real \(SU(5)\) representations which would have masses of \(0(10^{15} \text{ GeV})\).

Let us first see how hypothesis II) works. The \(\lambda = -1/2\) states in the \([2,\, [0]\) + \([5/2,\, [7]\) multiplets are in the following \(SU(8)\) representation

\[
8 + \overline{56} + \overline{246} + 504
\]

which, when decomposed with respect to \(SU(5) \times SU(2)\), can be written as a real part

\[
(45 + \overline{45}, \, d) + (24, \, 2 + 1 + 1) + \quad (2)
\]
\[(10 + \bar{10}, \ 3 + 2 + 2 + 2 + 1 + 1 + 1 + 1)\]
\[+\ 2(5 + \bar{3}, \ 2 + 1) + (1, \ 3 + 2 + 2 + 1 + 1)\]
\[+\ (\bar{5}, \ 2 + 1) + (\bar{5}, \ 2 + 1)\]

(2) cont.

and a complex part

\[(45, 2) + (40, 2 + 1) + (45, \bar{4}) + (10, \ 2 + 1)\]
\[+\ (5, \ 3 + 2 + 1 + 1)\]

so that the subset selected by hypothesis II) is the real part in (2) with the
\[(5, \ 2 + 1)\]
replaced by the \[(10, \ 2 + 1)\]
contained in the complex part. This gives three \((\bar{5} + 10)\) families of \(\text{SU}(5)\).

EGZ also claim that the scalars contained in the supercurrent multiplet are able
to break the \(\text{SU}(8)\) into \(\text{SU}(5)\) through the Higgs mechanism. But it is hard
to understand how to implement the Higgs mechanism since there is no renormalizable
\(\text{SU}(8)\) gauge theory to start with. Anyway, these scalars could only break \(\text{SU}(8)\)
into \(\text{SU}(5) \times \text{SU}(2) \times \text{U}(1)\), and the \(\text{U}(1)\) factor is not anomaly-free except
for real fermion representations. Therefore the Higgs mechanism is not enough
for the selection of the gauge group, which has to rely on hypothesis I).

The EGZ approach looks rather arbitrary in spite of the conspicuous results.
There is no reason to choose the largest gauge group or the maximum number of
fermion states; usually, simplicity in physics means minimizing the number of
ingredients in the model whenever some ambiguity remains. Last but not least,
the "cassation of the civil rights" of a whole set of left-handed states according
to the previous criteria also looks arbitrary.

Recently\(^{10}\), we have studied alternative approaches to that of EGZ. The
main difference is that we include in the effective GUT all the spin 1/2 fermions
of a superposition of supermultiplets \(\oplus \{\lambda, [k]\}\) which is selected according
to some natural criteria to be discussed below. In our first scheme, we make
the following assumptions:
a) each supermultiplet may appear once at the most in this superposition.
b) The \(\text{SU}(N)\) representation, denoted by \(F\), of the left-handed spin 1/2
    fermions in the superposition of supermultiplets is both complex and anomaly-
    free.
c) The purely complex part of $F$ has only totally antisymmetric $SU(N)$ representations, i.e.,

$$F = \sum_{k} b(k) \, [k]_{N} + \text{real } SU(N) \, \text{representation} \tag{4}$$

With these assumptions one can now define the effective GUT with $SU(N)$ gauge group for $N$ extended supergravity with fermions in the representation $F$. Since $N \leq 8$, one has to have the trivial embedding for $SU(N) \supset SU(5) \supset SU(3)$. Then from our assumption c),

$$F = n \left( 40 + \bar{5} \right) + SU(5) \, \text{real}$$
$$= SU(3) \, \text{real} \tag{5}$$

We now comment on these hypotheses. The single multiplicity requirement in a) is sufficient to have a non-trivial family generation, i.e., to avoid obtaining more families by repetition of a solution. The assumptions b) and c) have been formulated at the $SU(N)$ level without reference to any subgroup, since $SU(N)$ is the local symmetry at the supergravity level. Moreover, for the embedding $SU(N) \supset SU(n)$, with $N = n + (N - n) \frac{1}{2} \, (n \geq 3)$, the anomaly for an $SU(N)$ complex representation $R$ is equal to the anomaly calculated for the $SU(n)$ representations in the decomposition of $R$. Therefore, the physical requirement that colour $SU(3)$ [or $SU(5)$] is anomaly free for the fermion complex representation $F$ automatically implies that the whole $SU(N)$ is anomaly free as long as all states are retained. Instead, while our assumption c), expressed by Eq. (4), implies Eq. (5), the converse is not, in general, true. We will come back to this point later.

From our assumption a), we can show that the coefficients $b(k)$ in Eq. (4) satisfy the inequalities:

$$b(k) \leq \begin{cases} 2 & (N = 2n+1) \\ 4 & (N = 4n+2) \\ 4 & (N = 4n) \end{cases} \tag{6}$$

(depending on the representation $[k]$). Hypothesis b) can be expressed as

$$0 = \text{Anom} (F) = \sum_{k} b(k) \text{Anom} ([k]) = \sum_{k} b(k) \frac{N-k}{N-2} \binom{N-2}{k-1} \tag{7}$$

From (6) and (7) one then obtains the following results:
\[
\begin{array}{cccccc}
N & = & 5 & 6 & 7 & 8 \\
\text{Number of} & & \gg 6 & \gg 3 & \gg 14 & 15 \\
\text{supermultiplets} & & & & & \\
\{\mathbb{S}+10\} \text{ families} & n = & 2 & 4 & 2 & 3 \\
\end{array}
\]

(8)

Therefore, only \(N = 8\) supergravity gives three families in this scheme. The fermions in the effective \(SU(8)\) gauge theory are in the \(SU(8)\) representation \(F = 3 (56 + \overline{56} + \overline{5}) + \text{real}\). The \(SU(8)\) real part of \(F\) corresponds to particles with mass of \(O(M_P)\) that are to be naturally neglected in the effective GUT.

With respect to the \(SU(5)\) subgroup the three \(SU(8)\) families decompose into 3 \((5 + 10) + SU(5) - \text{real}\). The states in the real part are expected to have masses \(\geq 10^{15}\) GeV, the only light fermions being the three \((5 + 10)\) families.

Even if the dynamics of \(N\) extended supergravity is unknown and could be rather unconventional, one can obtain some additional constraints by assuming that the composites are normal bound states of the preons of the theory (graviton multiplet). On these grounds EGZ have observed a very general property related to the "N-ality" \(\nu\) of the \(SU(N)\) representations of these bound states. Indeed, the fermions in the graviton supermultiplet have odd \(\nu\), while the bosons have even \(\nu\). It is now obvious that this is a general property of conventional bound states of these preons, irrespective of the number of constituents. One can define a sort of parity \(\nu = (-)^\nu\), the allowed states being those with \((-)^\nu = (-)^{2\lambda}\), where \(\lambda\) is the helicity of the state. This rule is satisfied by all states in a supermultiplet \(\{\lambda, [K]\}\), provided

\[
(-)^k = (-)^{2\lambda}
\]

(9)

Unfortunately, the solutions obtained in the previous scheme \[\text{cf. Eq. (8)}\] for \(N = 6\) and \(N = 8\) do not satisfy such a rule. This means that our solution with three families for \(N = 8\) cannot be conventional bound states (this is not dramatic: remember that in the old quark model statistics were wrong ... and there were three good reasons for that!). We then try to weaken our assumptions a) and/or c). The latter can be modified by requiring non-exoticity at the \(SU(5)\) level rather than at the \(SU(N)\) level, i.e., by imposing Eq. (5) rather than the stronger condition in Eq. (4). In this case, Eq. (4) is replaced by
\[ F = \sum_{k=0}^{N-1} \sum_{\ell=1}^{N-1} b_{k\ell} \left( [k] \times [\ell] \right) \]  

(10)

and the single multiplicity condition a), together with the content (1) of supermultiplets, gives the constraints,

\[ b_{k\ell} \leq 2 \ (k \neq \ell) \quad , \quad b_{kk} \leq 1 \ (k \neq 0) \]  

(11)

and the \( b_{ko} \) satisfy the same conditions as \( b_k \) in Eq. (5). The hypothesis b) then reads:

\[ 0 = \text{Anom} (F) = \sum_{k=0}^{N-1} \sum_{\ell=1}^{N-1} b_{k\ell} \cdot \left\{ \frac{N-2k}{N-2} \binom{N-2}{k-1} \binom{N}{\ell} + \frac{N-2\ell}{N-2} \binom{N-2}{\ell-1} \binom{N}{k} \right\} \]  

(12)

For \( N = 6 \), we find that there is no solution of Eqs. (10) and (12) that satisfies the parity condition on \( N \)-alities, Eq. (9).

For \( N = 8 \), we find no solution to Eqs. (9), (10) and (12) with single multiplicity for the supermultiplets. At this point, our hypothesis a) has to be modified. Let us first see what happens if we impose no conditions on the supermultiplet multiplicities. From Eq. (9) (parity rule on \( N \)-alities), Eq. (10) (non-exoticity) and Eq. (12) (no anomaly) we find that any solution for \( F \) is a linear combination of three basic solutions

\[ F = \sum_{i=1}^{3} n_i F_i = 2n_3 \left( 5 + 10 \right) + \text{SU}(5) \text{ real} \]  

(13)

where

\[ F_3 = \left[ 3 \right] + 5 \left[ 7 \right] = 56 + 5(\overline{5}) \]

\[ = 2 \left( 5 + 10 \right) + \text{SU}(5) \text{ real} \]

and \( F_1, F_2 \) are two particular solutions which are real under the \( \text{SU}(5) \) subgroup. It is gratifying to notice that even if we have weakened our hypothesis c), the results we obtain are compatible with the more natural formulation of Eq. (4) if \( n_1 = n_2 = 0 \).
Without any restriction on supermultiplet multiplicities, one has many ways of obtaining Eq. (13) for any values of the \( n_i \)'s and thus of obtaining an arbitrary (even) number of families. Therefore, if the composites in the effective GUT are conventional bound states, one expects, on rather general grounds, that the number of fermion families in \( N = 8 \) supergravity is even!

Therefore, fermion family generation in \( N = 8 \) supergravity requires some possibly physical restriction on the supermultiplet multiplicities. A reasonable assumption that we have explored is that composites of the effective GUT are two- preon bound states. We then consider the massless limit of an irreducible massive representation of \( N = 8 \) supersymmetry which is bilinear in the components of the graviton supermultiplet \( \{2, [0] \} \) [two-particle representations of supersymmetry are studied in Ref. 7]:

\[
\left( \{2, [0] \} \times \{2, [0] \} \right)_L \equiv \sum_{J_{\text{MAX}}} J_{\text{MAX}} = 4 + L, [0]_{\frac{3}{2}}^3
\]

\[
\sum_{m \to 0} \sum_{h=a}^{L} \sum_{k=0}^{4} \left\{ 4 + h - \frac{k}{2}, [k] \right\} + \sum_{h=0}^{L} \left\{ 2 + h, [4] \right\}
\]

where the irreducible massive supermultiplet is characterized by the internal angular momentum \( L = 0, 2, 4, \ldots \) [see Ref. 7] [corresponding to an SU(\( N \)) singlet state with maximum spin \( J_{\text{MAX}} = 4 + L \). We find in particular that if the gauge bosons are obtained in the \( n \to 0 \) limit of a massive spin one state in the adjoint representation of \( SU(\( N \)) \), they have to be either in the \( L = 2 \) massive supermultiplet or in the \( L = 4 \) one. It could then be reasonable to assume that the light states are all to be found in the \( L = 0, 2 \) and \( 4 \) massive supermultiplet. But in this case one gets only two fermion families. One can obtain two additional families by also including \( L = 6 \) for instance.

Let us recall the general philosophy in these attempts at superunification. The starting point is the common hope that extended supergravity is a serious candidate for a superunified theory, which should comprise the GUT as an effective theory. The only conceivable possibility in the framework of \( N \leq 8 \) supergravity is that the particles in the effective GUT are composite states, which, in turn, are components of some representations of supersymmetry. Since the dynamics of extended supergravity is far from being known, the strategy up to now has been to identify the effective GUT by assuming some reasonable rules which could allow for non-trivial fermion family generation. The hope is to find some systematics in the results which could suggest a more fundamental origin of the solutions.
Unfortunately, we find that if the possible patterns of fermion family generation look promising, there seems to be no simple systematics in the set of supermultiplets selected by the rules of the game. Perhaps these rules are too naive. It might be that the dynamics of supergravity does not exhibit the kind of systematics and selection rules we are used to.

REFERENCES

1) For an extensive review of GUTs and references to the original work see, e.g., L. Okun, "Quarks and Leptons", ed. "Nauka", Moscow (1961) ch. 25 (in Russian).

2) For a review of supersymmetry (and original references) see, e.g., P. Fayet and S. Ferrara, Phys. Rep. 32 (1977) 249.

3) See, e.g., E. Witten, Princeton preprint (1981). See also Ref. 2.


