THE \((56,1^3)\) BARYONIC MULTIPLE AND THE SPIN-AVERAGED MASS OF THE \(\Delta\)-RESONANCE SECTOR

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ABSTRACT

By considering spin-averaged masses of \(\Delta\) resonances from the \(n = 0,1,2\) harmonic oscillator multiplets of three quarks, we show that sum rules can be easily derived predicting the spin-averaged mass of the \(\Delta\) resonance sector of \((56,1^3)\). This prediction depends only on spin-singlet symmetry-breaking operators and so the sum rules are independent of questionable assumptions about spin-vector and spin-tensor operators. Using only data from experiment, the prediction is found to be on average 125 - 150 MeV higher than the spin-averaged mass of the experimental \(\Delta\) states plausibly assigned to the \((56,1^3)\).
1. INTRODUCTION

The three-quark harmonic oscillator model of baryons has been very successful in providing a framework within which the properties of baryon resonances may be discussed and classified. It is therefore important to isolate possible areas where the quantitative predictions of the model unambiguously disagree with experiment. One such area of disagreement is the prediction of the mass of the \( \Delta D_{35} \) state belonging to the \((56, l_1)\) multiplet. The mass of this state is related by a sum rule to the parameters of fits to the masses of the \( n = 0,1,2 \) oscillator excitations. Dalitz et al.\(^1\) predicted a mass of \( 2088 \pm 25 \text{ MeV} \) for this state, whereas the latest partial wave analyses\(^2\) give the experimental mass in the range \( 1900 - 1950 \text{ MeV} \). These two figures disagree on average by over 100 MeV and point to a quantitative failure of the model. However, in a recent letter by Bowler et al.\(^3\), a new analysis of this problem is presented, using an approach based on the \( \text{Sp}(12) \) spectrum generating group of the three-quark harmonic oscillator. They calculate the mean mass of the non-strange sector of the \((56, l_1)\) to be about \( 1985 \text{ MeV} \). Because this is close to the measured mass of the \( \Delta D_{35}(1925) \) they conclude that, given the simplicity of their model and their neglect of hyperfine interactions, this is startlingly good agreement. They argue that the prediction of Dalitz et al.\(^1\) should be superceded by their estimate since the \( \text{SU}(6) \) approach of Dalitz et al. assumed that spin-orbit forces were relevant and totally neglected spin-tensor terms. The work of Isgur and Karl\(^4,5\), which has had generally phenomenological success in describing the baryon mass spectrum, propounds the opposite view, namely that spin-orbit forces are, for some reason, negligible, and that spin-spin and spin-tensor forces as described by single-gluon exchange are dominant. Bowler et al.\(^3\) argue that this invalidates the prediction of Dalitz et al.\(^1\) and that their (i.e. Bowler et al) derived mass value is more likely to be correct, since they base their analysis on the later, successful, mass fits of Isgur and Karl\(^4,5\). However, as later argued, this statement is not correct. We would certainly expect the mass of the \( \Delta D_{35} \) to lie above the predicted average mass of the non-strange sector of the \((56, l_1)\). This worsens the approximate agreement cited by Bowler et al.\(^3\). We show that the mass value predicted by Dalitz et al. is substantially correct given the harmonic oscillator description of baryon multiplets. The only ambiguity of that work concerns the exact nature of the forces which split the \( \Delta \) resonances within a given submultiplet (e.g. \( \text{SU}(3) \otimes \text{SU}(2) \) of \( \text{SU}(6) \)). Data show that such states are more or less degenerate in mass [see e.g. \((56, l_2)\)], a result used by Dalitz et al\(^1\) to show that the coefficient of spin-orbit splitting between the \((56, l_2^*)\) \( \Delta \) states is small (they quote \( 2 \pm 12 \text{ MeV} \)). The ambiguity concerning the effect of hyperfine interaction then has only a very small effect on the final result for
the $\Delta D_{35}$ mass. There should, therefore, be very little difference between approaches to this problem based on the Isgur and Karl\textsuperscript{4,5} description of the baryon spectrum or on that of Dalitz et al.\textsuperscript{13}. We show that this is the case in the next section. We show how sum rules may be constructed for spin-averaged quantities which are insensitive to hyperfine interactions. We derive a prediction for the spin-averaged mass of the $\Delta$-sector of the $(56,1_3^-)$ and compare our prediction with the experiment: we find our prediction is too high by about 150 MeV. We then discuss the average mass of the non-strange sector of the $(56,1_3^-)$ and find that the estimate of Bowler et al.\textsuperscript{3) is slightly too low by about 30 MeV on average. A comment then follows about the absence of exchange forces in the baryon spectrum and the constraint that this places on sum rules in general.

2. SUM RULES FOR AVERAGED QUANTITIES

Consider any term in the Hamiltonian which breaks the degeneracy of the harmonic oscillator and which transforms as other than a scalar under some subgroup $\mathcal{H}$ of the full classification group $G$. This operator is traceless with respect to the trace over indices of the subgroup $\mathcal{H}$. Such a symmetry-breaking mass operator cannot therefore contribute to the mean mass of any given irreducible representation of $\mathcal{H}$\textsuperscript{1,6}, i.e.

$$
M(\mathcal{H}) \equiv \frac{\sum_{h \in \mathcal{H}^3} M(h) \cdot d_h}{\sum_{h \in \mathcal{H}^3} d_h}
$$

(1)

is independent of the contribution of such mass operators\textsuperscript{4).} $M(h)$ is the mass of the state $\mathcal{H}(h)$ where $\mathcal{H}$ is the set of all states forming an irreducible representation of $\mathcal{H}$ and $d_h$ is the degeneracy of $h$ (i.e. with respect to the full classification under $G$) within $\mathcal{H}$. In particular if we average over the spins of a given species of baryon (i.e. given isospin, strangeness and submultiplet assignment), the resulting mean mass can only depend on spin-scalar operators, i.e. operators that transform as scalars under full SU(2) rotation, e.g. $\hat{S}_I \cdot \hat{S}_J$. Consequently if we discuss only the average mass of the $\Delta$ resonances from the relevant 56-plets, our calculations are not sensitive to any ambiguity arising from the parametrisation of the spin-splitting forces. Thus one can do better than Bowler et al.\textsuperscript{3) who only predict the average mass of the strangeness zero sector of the $(56,1_3^-)$ multiplet, whereas the average mass of the $\Delta$ sector alone can easily be computed. This result is inherent in Eq.(8)

\textsuperscript{4) If mixing between different irreducible representations is considered, then the sum should be over all states of the representations involved.
of Ref. 1), where the spin-orbit term can be neglected (in agreement with Isgur and Karl4,5) - rewriting this equation more rigorously in terms of spin-averaged masses eliminates the spin-orbit contribution and immediately gives Eq. (3b) (see below). From Eq. (1) we define the spin-averaged baryon masses:

$$\langle B \rangle = \frac{\sum (2J+1) B(J)}{\sum (2J+1)}$$

(2)

where $B(J)$ represents the mass of a baryon of species $B$ and spin $J$ belonging to a particular $SU(3) \times SU(2)$ multiplet. These averaged masses only depend on spin-scalar mass operators and from Eq. (4) of Ref. 1 we have that for any such operator $V_0$:

$$\langle 56, 1^3 | V_0 | 56, 1^3 \rangle = \frac{3}{2} \langle 56, 2^+ | V_0 | 56, 2^+ \rangle + \frac{5}{6} \langle 56, 0^+ | V_0 | 56, 0^+ \rangle$$

$$- \frac{1}{6} \langle 56, 0^+ | V_0 | 56, 0^+ \rangle$$

(3a)

This equation is derived purely on the basis of the harmonic oscillator description of the states1,6), and is independent of the classification group $G$. Since the average mass of the $\Delta$ sector of the 56-plet is expressed in terms of the same linear combination of all spin-singlet operators irrespective of the $O(3)$ classification7), we find immediately that

$$\langle \Delta | 56, 1^3 \rangle = \frac{3}{2} \langle \Delta | 56, 2^+ \rangle + \frac{5}{6} \langle \Delta | 56, 0^+ \rangle - \frac{1}{6} \langle \Delta | 56, 0^+ \rangle$$

(3b)

Enough data exist to determine the right-hand side of Eq. (3b) without recourse to mass fitting in order to predict relevant mixing masses. From the Particle Data Group2) we find

$$\langle \Delta | 56, 2^+ \rangle = 1880 - 1930 \text{ MeV}$$

$$\langle \Delta | 56, 0^+ \rangle = 1670 - 1690 \text{ MeV}$$

$$\langle \Delta | 56, 0^+ \rangle = 1232 \text{ MeV}$$

(4)
which leads to:

\[
\langle \Delta (56, \frac{1}{2}^+) \rangle \approx \text{pred.} 2029 - 2079 \text{ MeV}
\]

The errors on the quoted masses are encoded by giving the two extreme values of the average masses. We assumed that the \( \Delta D_3 \) state in the \((56, \frac{1}{2}^+)\) lay in the neighbourhood of 1910 MeV - in any case this state is not heavily weighted in the averaging procedure compared with \( \Delta E_{\frac{3}{2}} \). From the Particle Data Tables\(^2\) and from Ref. 8) we find candidates for all the \( \Delta \) resonances in the \((56, \frac{1}{2}^+)\), namely \( \Delta D_3 \) (1960), \( \Delta E_{\frac{3}{2}} \) (1900) and \( \Delta D_3 \) (1920). The last resonance is new\(^3\) and needs confirmation, but its mass is compatible with the masses of the other states for this assignment. We then find

\[
\langle \Delta (56, \frac{1}{2}^+) \rangle \approx \text{exp.} 1880 - 1930 \text{ MeV}
\] (5)

with mean value at 1905 MeV.

This is to be compared with the mean value of the range computed above, namely

\[
\langle \Delta (56, \frac{1}{2}^+) \rangle \approx \text{pred.} 2055 \text{ MeV}
\] (6)

This is close to the estimate 2088 \( \pm 25 \) MeV of Dalitz et al.\(^1\) and shows that Bowler et al.\(^3\) should have estimated how far above the average mass of the non-strange sector the \( \Delta \) resonances of the \((56, \frac{1}{2}^+)\) lie. We remark that we have used only the harmonic oscillator and the Particle Data Tables\(^2\).

In order to make even clearer contact with the result of Bowler et al., we can apply a relation analogous to those shown in Eq. (3) to the central masses of the multiplets concerned. This has already been done in Ref. 6) where, however, the central masses of complete multiplets including strange particles, are given. From Ref. 6) we obtain

\[
\langle 56, \frac{1}{2}^+ \rangle = 1750 \text{ MeV}
\]

Of course this value does not involve any assumptions about spin non-singlet forces.

Regarding the work of Isgur and Karl\(^4,5\) we note that one important feature is that generalized Pauli statistics are only applied to the non-strange components of any state. This suggests that an SU(4) \( \otimes \) SU(2) classification of baryons is more appropriate than the usual SU(3) basis. The SU(4) corresponds to the spin-isospin transformation of the non-strange quarks and the SU(2) corresponds
to the strange quark spin transformations. Of course, such a choice does not
invalidate our previous arguments concerning the $\Delta(56,1^-_I)$ average mass, since
these arguments were independent of the group $G$. However, we can restrict our-
sewrems to the strangeness-zero sector of the multiplets (as considered in Refs. 4,5)
i.e., to the symmetric 20-plets of an SU(4) classification. We use that

$$
\langle \mu, L_n^{p} \rangle = \frac{\sum_{\mu} (2J+1)(2I+1) M(J,I)}{\sum_{\mu} (2J+1)(2I+1)}
$$

where $\sum_{\mu} (2J+1)(2I+1) = D_{\mu}^L$, the multiplet dimension, $M(J,I)$ is the mass
of a state of spin $J$ and isospin $I$. A relationship equivalent to those in
eq(3) still holds, and from the Particle Data Tables alone and considering only
the non-strange members of the relevant SU(6) multiplets, we find (with SU(4)
labelling):

$$
\langle 20, 2_t^+ \rangle = 1840 - 1860 \text{ MeV (1850 MeV)}
$$
$$
\langle 20, 0_o^- \rangle = 1620 - 1645 \text{ MeV (1600 MeV)}
$$
$$
\langle 20, 0^+_o \rangle = 1173 \text{ MeV}
$$

(8)

The numbers in brackets are those given by Isgur and Karl in Ref. 5). Of course
these numbers are lower than those quoted in Ref. 6) for the full multiplets, since
the strange sector has been excluded here. We then derive

$$
\langle 20, 1^-_3 \rangle = 1490 - 2038 \text{ MeV (1980 MeV)}
$$

The value in brackets derived from the Isgur and Karl figures is close to that
quoted by Bowler et al. 3) of 1985 MeV.

We therefore disagree with the result of Ref. 3) and conclude that the har-
monic oscillator prediction for the average mass of the $\Delta$ resonances from the
$(56,1^-_I)$ is, on average, about 150 MeV higher than the experimentally observed
value of 1905 MeV. As a by-product, we predict

$$
\langle N(20, 1^-_3) \rangle = 1802 - 1883 \text{ MeV}.
$$

There are no candidates for NS111 or ND13 in the region (except perhaps the
one-star ND13(1900)). Not for this prediction nor for the prediction of the
$[\Delta(56,1^-_I)]$ average mass were any assumptions about spin non-scalar forces
required. We emphasize that it is the harmonic oscillator assumption alone which is being tested. The inability of the harmonic oscillator model to incorporate the most likely $\Delta$-resonance candidates into the $(56,1^+_3)$ multiplet requires either the constraints imposed by the oscillator model to be relaxed, or the postulate that new states not included in this model exist. The harmonic oscillator is unlikely to be the basis for an exact theory of the baryon spectrum, so the former suggestion should be seriously considered. However, the classification of the relevant $\Delta$-resonances must be studied further before conclusions concerning their nature are unambiguously drawn.

If we wish to take into account the mixing between the 56- and 70-plets for $L_\pi = 0^+_2, 2^+_2$, then the spin-averaged $\Delta$ masses must be calculated using states from both the 56-plets and 70-plets. The shift in the masses because of this supermultiplet mixing then cancels out in this average. From Table 1 of Ref. 6, we then find the slightly more complicated sum rule

$$\langle \Delta (56, 1^+_3) \rangle = \frac{4}{15} \left[ 3 \langle \Delta (56, 2^+_2) + \Delta (70, 2^+_2) \rangle - \langle \Delta (70, 1^+_1) \rangle \right] + \frac{5}{6} \langle \Delta (56, 0^+_2) \rangle - \frac{11}{30} \langle \Delta (56, 0^+_0) \rangle$$

(9)

From the Particle Data Tables 2):

$$\langle \Delta (70, 1^+_1) \rangle = 1640 - 1710 \text{ MeV}$$

$$\langle \Delta (70, 2^+_2) \rangle = 1975 \text{ MeV}.$$

The average $\Delta$ mass from $(70, 2^+_2)$ is estimated from previous fits 4,9). This gives

$$\langle \Delta (56, 1^+_3) \rangle = 2013 - 2075 \text{ MeV}$$

The error is large mainly because of the poor determination of the $\Delta D_{33}$ mass in $(70, 1^+_1)$. Photoproduction gives low values around 1650 MeV for this state, whereas pion-nucleon scattering gives values around 1700 MeV or higher. Nevertheless, this estimate is compatible with the estimate based on Eq. (3) - it is higher on average than the experimental value of 1905 MeV by some 150 MeV.

A brief comment is in order concerning the anharmonic potential introduced by Bowler et al. in their Hamiltonian, and earlier suggested by Isgur and Karl 4,5).
This potential does not distinguish between states which have even and odd space
permuation symmetry. An exchange potential could make this distinction, but this
has not been included - certainly QCD would suggest that such a term was absent
(in perturbation theory at least) - since the quark colour current is a scalar
under SU(3) flavour transformations. The choice of Bowler et al. leads to the
relationships between multiplets quoted by them and which are inherent in Table
1 of Ref. 6) if \( a_n = b_n \). The coefficients \( a_n \) and \( b_n \) distinguish the two sepa-
rate contributions to the central mass of a multiplet coming from the space per-
mutation of even and odd two-body components of the states respectively. That is

\[
\begin{align*}
\alpha_n &= 4\pi \int R_n x^{2n} V_A(x^2) \exp(-x^2 x^2) \, dx \\
\beta_n &= 4\pi \int R_n x^{2n} V_B(x^2) \exp(-x^2 x^2) \, dx \\
R_n^{-1} &= \int_0^\infty x^{2n} \exp(-x^2 x^2) \, dx
\end{align*}
\]

(9)

where the potential between quarks \( i \) and \( j \) in the Hamiltonian is

\[
V_A(\tau_i^2) \{ T_i^{\dot{\imath} \dot{\jmath}} S_i^{\dot{\imath} \dot{\jmath}} + T_o^{\dot{\imath} \dot{\jmath}} S_o^{\dot{\imath} \dot{\jmath}} \}
+ V_B(\tau_j^2) \{ T_i^{\dot{\imath} \dot{\jmath}} S_i^{\dot{\imath} \dot{\jmath}} + T_o^{\dot{\imath} \dot{\jmath}} S_o^{\dot{\imath} \dot{\jmath}} \}
\]

(10)

where \( T_{ij}^{\dot{\imath} \dot{\jmath}}(S_{ij}^{\dot{\imath} \dot{\jmath}}) \) are isospin (spin) projection operators onto states of isospin
(spin) \( J \) of quarks \( i \) and \( j \).

\[
T_i^{\dot{\imath} \dot{\jmath}} = \left( \frac{3}{4} + \tau_i \cdot \tau_j \right), \quad T_o^{\dot{\imath} \dot{\jmath}} = \left( \frac{1}{4} - \tau_i \cdot \tau_j \right)
\]

(11)

and similarly for \( S_{ij}^{\dot{\imath} \dot{\jmath}} \).

If \( V_A(x^2) \neq V_B(x^2) \) then exchange forces are present, and the relationships
between the 70-plet and 56-plet inferred from the equations in Bowler et al. do
not hold. The assumption \( V_A(x^2) = V_B(x^2) \) can be tested easily in one case
since it implies \( a_4 = b_4 \). From Table 1 of Ref. 5), we see (in the SU(4) model)

\[
\begin{align*}
\alpha_4 &= \frac{5}{3} \left< 20, 2^+_2 \right> - \frac{2}{3} \left< 20, 0^+_2 \right> \\
b_4 &= 2 \left< 20, 1^-_1 \right>_M - \left< 20, 0^+_o \right>
\end{align*}
\]
20\textsubscript{M} is the symbol for the mixed symmetric non-strange sector of the more familiar 70-plet of SU(6). From data,

\[ \langle 20, 1^+ \rangle_\text{M} = 1613 - 1688 \text{ MeV} \]

From this and previous results we get

\[ a_4 = 1970 - 2053 \text{ MeV} \]
\[ b_4 = 2053 - 2163 \text{ MeV} \]

These are not in bad agreement - a difference of 80 MeV on average. This upholds the assumption that the mean mass operator is independent of exchange forces. It must be remarked, however, that the results deduced above for the \( \Delta(56,1^3) \) average mass are independent of this assumption, since all masses used in applying Eq. (3) were associated only with the fully space-symmetric sector of either SU(4) or SU(6); i.e., no data from an SU(4) 20\textsubscript{M} (670 of SU(6)) were required.

The validity of Eq. (9) is also independent of assumptions about exchange forces, since the \( b_4 \) coefficient cancels between \( \Delta(70,2^+_2) \) and \( \Delta(70,1^-_1) \).

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REFERENCES


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