GAS AMPLIFICATION VARIATIONS OF PROPORTIONAL COUNTERS

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The amplification of such a device is dependent upon the following parameters: high tension, gas density, and geometrical configuration (gas composition is ignored). It will be shown that there exist relations between these parameters which might be of interest for designers of large proportional counter arrays.

It is assumed that there is true linear amplification, i.e. that space-charge effects are negligible. Therefore, the gas amplification can simply be expressed by

\[ \ln A = \int_{r_1}^{d} \alpha ds, \]

where \( s \) is the path of the electron between its locus of production \( d \) and its terminus at the sense wire \( r_1 \).

The first Townsend coefficient \( \alpha \) can be considered quite generally as:

\[ \alpha = \rho f \left( \frac{E}{\rho} \right), \]

with \( E \): electr. field strength; \( \rho \): gas density.

For \( E \) one can write

\[ E = U g(x), \]

with

\[ U: \text{ high voltage applied to the counter}; \]
\[ g(x): \text{ function, depending on the geometry of the counter.} \]

Because only the region very near to the wire contributes essentially to the amplification, the upper integration limit can also be chosen to be at the cathode \( R \).

**Variation due to voltage and density**

The Taylor expansion of the Townsend coefficient gives:

\[ \alpha = \rho f + U f' g \frac{dU}{U} + (\rho f - U f' g) \frac{d\rho}{\rho}, \]

where

\[ f' = \frac{\partial f}{\partial (E/\rho)}. \]
Therefore,
\[
\beta = \int \frac{R}{r_1} \rho \int f' g \, ds = \rho \int f' g \, ds + \frac{\Delta U}{U} \int U f' g \, ds + \frac{\Delta \rho}{\rho} \int (\rho f' - U f' g) \, ds
\]
\[
= \beta_0 + K_1 \frac{\Delta U}{U} + K_2 \frac{\Delta \rho}{\rho},
\]
where
\[
K_1 = U \int f' g \, ds, \quad K_2 = \int (\rho f' - U f' g) \, ds = \beta_0 - K_1.
\]

For cathodes of polygonal shape the potential distribution can be approximated sufficiently well for our purposes by that of a coaxial structure. (The multipole terms of the development give rise to a correction of \(< 10^{-6}\) in the vicinity of the sense wire; see Appendix.) Therefore \(g\) can be approximated by
\[
g = \frac{1}{\ln (R/r_1)} \frac{1}{r}.
\]

\(R\), the "outer radius" of the structure, can be determined by the multipole development, and it turns out that it is an "average radius" of the polygon.

The amplification under a variation of voltage is given below:
\[
\beta = \int \frac{R}{r_1} \rho \left[ U \left( 1 + \frac{\Delta U}{U} \right) \right] \ln \left( \frac{r}{r_1} \right) \, dr
\]
\[
= \left( 1 + \frac{\Delta U}{U} \right) \rho \int_{r_1'}^{r'} f \left( \frac{U}{r' \ln \left( \frac{R}{r_1} \right)} \right) \, dr',
\]
where:
\[
r' = r \left( 1 - \frac{\Delta U}{U} \right).
\]
The integration yields

\[
\beta = \left(1 + \frac{\Delta U}{U}\right) \left[\beta_0 + \rho r_1 f(r_1) - R f(R)\right] \frac{\Delta U}{U},
\]

where

\[
f(r) = \int \frac{U}{r \ln \frac{R}{r_1}} dr.
\]

For proportional counters the following inequality can always be assumed:

\[r_1 f(r_1) \gg R f(R).\]

The last expression for \(\beta\) can be simplified as follows:

\[
\beta = \beta_0 + \left[\beta_0 + \rho r_1 f(r_1)\right] \frac{\Delta U}{U}.
\]

A comparison with Eq. (1) shows

\[
K_1 = \beta_0 + \rho r_1 f(r_1).
\]

**Amplification under variation of sense wire radius**

\[
\beta = \int_{r_1 + \Delta r_1}^{R} \rho f\left(\frac{U}{r_0 \ln \frac{R}{r_1 + \Delta r_1}}\right) dr
\]

\[
= \left(1 + \frac{1}{\ln \frac{R}{r_1}}\right) \rho \int_{r_1 + \Delta r_1}^{R'} f\left(\frac{U}{r_0 \ln \frac{R}{r_1}}\right) dr',
\]

with

\[
r' = r\left(1 - \frac{1}{\ln \frac{R}{r_1}}\right)\frac{\Delta r_1}{r_1}.
\]

Using the same approximation as before, we obtain

\[
\beta = \beta_0 + \left(K_2 + \frac{K_1}{\ln \frac{R}{r_1}}\right) \frac{\Delta r_1}{r_1}.
\]

**Amplification under variation of outer radius**

The same procedure is applied as for the sense wire; therefore only the result will be given:

\[
\beta = \beta_0 - \frac{K_1}{\ln \frac{R}{r_1}} \frac{\Delta R}{R}.
\]

(5)
Amplification for sense wire displaced from the centre of symmetry

In this case the contribution of the dipole term might no longer be negligible (see Appendix). The largest error occurs for an electron, produced at \( \phi = 0 \):

\[
\beta = \rho \int_{r_1}^{R-\Delta} \frac{U}{r} \ln \left( \frac{R}{r_1} - \frac{\Delta}{R} \right) \left( \frac{1}{r} + \frac{\Delta^2}{R^2} + \frac{dr^2}{r^2} + \frac{1}{r^2} \right) dr
\]

\[
= (1 + \varepsilon) \rho \int_{r_1}^{r'} \frac{U}{r} \ln \left( \frac{R}{r_1} - \frac{\Delta^2}{R^2} + \frac{a + b}{r^2} \right) dr',
\]

where

\[
r' = r(1 - \varepsilon); \quad a = \frac{\Delta}{R^2} (1 - \varepsilon); \quad b = \frac{\Delta^2}{R^2} (1 - \varepsilon); \quad \varepsilon = \left( \frac{\Delta}{R} \right)^2 \frac{1}{\ln \frac{R}{r_1}}.
\]

We put

\[
E = \frac{U}{\ln \frac{R}{r_1}} \cdot \frac{1}{r'}; \quad \Delta E_1 = \frac{U}{\ln \frac{R}{r_1}} a; \quad \Delta E_2 = \frac{U}{\ln \frac{R}{r_1}} b \cdot \frac{1}{r'^2}.
\]

Herewith the Taylor expansion of the integral:

\[
0 \int f(E + \Delta E) dr' = \rho \int f(E) dr' + a \cdot \frac{U}{\ln \frac{R}{r_1}} \int f' dr' + b \cdot \frac{U}{\ln \frac{R}{r_1}} \int f' \frac{1}{r'^2} dr',
\]

\[
= \text{I} + \text{II} + \text{III}
\]

where

\[
f' = \frac{\partial f}{\partial (E/E)},
\]

The first integral can be treated with the same method as before, and its contribution to the amplification is

\[
\beta = \beta_0 + \left( \frac{\Delta}{R} \right)^2 \frac{K_1}{\ln \frac{R}{r_1}}.
\]

To evaluate the second term, we approximate \( f(E) \) in the vicinity of the wire by

\[
f(E) = f(E_0) + C(E - E_0)^n.
\]
Remembering that only the region $10r_1 > r > r_1$ contributes to the amplification, it can be shown by simple integration that the second term yields
\[ \Delta R_{II} \leq 10\beta_0 \frac{r_1}{R^2} \quad \text{for} \quad -9 \leq n \leq 0 . \]

In the third integral we put
\[ f' = \frac{\partial f}{\partial r'} \frac{\partial r'}{\partial R} , \]
and get
\[ b \frac{U}{\ln \frac{R}{r_1}} \int_{r_1}^{R} \frac{1}{r'^{1/2}} \, dr' = -pb \int_{r_1}^{R} \frac{\partial f}{\partial r'} \, dr' = \rho \cdot bf(r_1) . \]

Considering also Eq. (3), we obtain:
\[ \Delta R_{III} = \frac{\Delta R_1}{R^2} (K_1 - \beta_0) . \]

Finally,
\[ \beta = \beta_0 + \frac{K_1}{\ln \frac{R}{r_1}} \left( \frac{\Delta R}{R} \right)^2 + \frac{K_1 + 9\beta_0}{K_1} \frac{\Delta R_1}{R^2} . \quad (6) \]

The parameter $K_1$ can be measured simply by a voltage variation; $K_2$ is obtained by a pressure variation, provided photoelectrons with a very short range have been used as an ionization agent. If minimum ionizing particles are used, it must also be taken into account that the number of initially produced electrons is proportional to the pressure. The measured parameter $\bar{K}_2$ is related to $K_2$ by
\[ \bar{K}_2 = K_2 + 1 . \]

With the help of $K_1$, $K_2$, and Eq. (2), it is possible to work out from Eqs. (4)-(6) the amplification variations due to geometrical errors.
APPENDIX

The appropriate development of the two-dimensional Laplace equation is the multipole one. A solution which fits the boundary conditions on the surface of the sense wire is given by

\[
U = K \left\{ \ln \frac{r}{r_1} + \sum_{n=1}^{\infty} a_n \left[ \left( \frac{r}{r_1} \right)^n - \left( \frac{r_1}{r} \right)^n \right] \cos n\phi \right\}
\]

For a cathode of square cross-section of width \( R \) a rather good solution is obtained by taking into account two terms only:

\[
U = \frac{U_0}{\ln \frac{r}{r_1}} \left\{ \ln \frac{r}{r_1} + 7 \times 10^{-2} \left( \frac{r_1}{R} \right)^6 \left[ \left( \frac{r}{r_1} \right)^4 - \left( \frac{r_1}{r} \right)^4 \right] \cos 4\phi \right\}
\]

The field strength in the vicinity of the wire, where amplification takes place, is sufficiently well determined by the first term only.

For a cylindrical structure, where the sense wire is displaced from the symmetry centre by \( \Delta r \), a sufficiently good solution is found with:

\[
U = \frac{U_0}{\ln \frac{r}{r_1} - \left( \frac{\Delta r}{R} \right)^2} \left[ \ln \frac{r}{r_1} + \frac{\Delta r_1}{r_1} \cos \phi \right]
\]