A WIDE BAND TRAVELLING WAVE ACCELERATING METHOD
FOR A 300 GeV PROTON SINCHROTRON

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I. INTRODUCTION

For the 300 GeV proton synchrotron proposed by the CERN Study Group (1) a peak accelerating voltage of 13.4 MeV per turn is required with 45° stable phase angle. The injection energy is 8 GeV so that the frequency variation during acceleration is only 0.55%. This frequency swing is so small that fixed-tuned wide-band accelerating structures become advantageous in spite of the large required voltage.

The proposed accelerating system, which is described in this paper, consists of wide-band travelling-wave structures which, as will be explained below, require substantially less power than comparable structures made of resonant cavities.

II. PRINCIPLE

It is not difficult to construct a periodically loaded travelling wave structure whose bandwidth greatly exceeds the 0.55% required frequency variation. The difficulty is, however, that the guide wavelength tends to vary rapidly with frequency whereas, ideally, a constant guide wavelength would be required for synchrotron acceleration.

Inside the pass band the guide wavelength cannot be made independent of frequency, since the rate of change of frequency with inverse guide wavelength is the group velocity which cannot be made greater than the velocity of light c. In fact, in order to arrive at an efficient accelerating structure, it is desirable to make the group velocity as small as possible since the power flow required to produce a given accelerating field is inversely proportional to group velocity. Nevertheless, an acceptable compromise can be found whereby an error of guide wavelength and a corresponding amount of phase slip between particle and wave is tolerated in order to maximize the efficiency of the structure.

It is advantageous to make the phase velocity \( v_p \) of the travelling wave structure equal to the particle velocity at the centre of the frequency band \( \Delta f \). Under this condition the optimum group velocity \( v_g \), leading to maximum total ener...
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Energy gain along the structure of length $z$ at the edges of the frequency range, can be shown \((1, 2)\) to be given by

$$v_g = \frac{\pi \Delta f}{2.34}$$ \[1\]

It turns out, as a consequence of eq. \([1]\), that the guide length is always short compared with the attenuation length, even if structures with rather low Q-factors are used. Thus, almost all the input power is dissipated in the terminating load and the accelerating field is nearly constant along the guide.

Under the condition of eq. \([1]\) the maximum energy gain $U$ of a particle traversing the structure at the edges of the frequency range $\Delta f$ is given by

$$U^2 = \frac{2.9 R f}{P}$$ \[2\]

where $f$ is the central frequency, $P$ is the input power and $R/Q$ the ratio of shunt impedance per unit length over the Q-factor, is given by

$$R = \frac{(accelerating\ field)^2}{Q \omega \times \text{stored\ energy\ per\ unit\ length}}$$ \[3\]

The quantity $R/Q$ is proportional to frequency. Otherwise it is a constant which depends only on the geometry of the structure.

If, instead of the travelling wave structure, fixed-tuned resonant cavities of the same $R/Q$ were used, one would find their performance expressed by an equation identical to eq. \([2]\) except that the numerical factor 2.9 would be replaced by unity and $P$ would stand for the reactive input power required at the edges of the frequency band. Hence, the travelling-wave guide is superior to the resonant cavity by at least a factor of 2.9. In fact, the superiority of the travelling wave method is even greater since, in the resonant cavity case, all the dc input power to the final amplifier is dissipated in the anode of the tube whereas the anode dissipation in the travelling wave case is only $1 - \eta$ times the input power where $\eta$ is the efficiency of the amplifier.

The proposed accelerating frequency of the 300 GeV synchrotron is 180 MHz. At that frequency effective values of $R/Q$ of about 560 $\Omega/m$ have been attained so far (cf. Section IV). It is intended to employ accelerating guides of about 14 m length each, since two guides of this length will fill the free space of one long straight section and each individual guide can be powered by one of the largest available r.f. power sources. The optimum group velocity becomes 0.062 c in this case but the optimum is quite uncritical, the 90% limits being at 0.088 c and 0.048 c. Table I gives a list of the proposed r.f. parameters.

III. BEAM LOADING

The presence of beam loading modifies the spatial distribution of electric field along the guide so that the guide output power no longer equals the input power. The change of input power required to restore the average accelerating field to its desired value does not necessarily equal the beam power but depends on the value of the stable phase angle and of the phase-slip between particle and wave i.e. on the frequency deviation from the centre frequency.

At the centre frequency, when the phase velocity equals the particle velocity, the average beam-induced field in the guide is 180° out of phase with the beam, i.e. purely decelerating.

However, during most of the machine cycle the frequency is very close to either the minimum frequency (injection period) or to the maximum frequency. Under these conditions, and if the group velocity has the value given by eq. \([1]\), the average beam induced field can be shown to be 44° out of phase with a purely decelerating field. The angle is leading when the frequency is below the centre value and lagging when the frequency is above.

Since the value of 44° is very close to the chosen nominal value of the stable phase angle and since — above transition — the total accelerating wave must have a leading phase angle with respect to the bunches, it follows that the beam induced field is very nearly 90° out of phase with respect to the desired field for all energies above the transition energy. In this case, compensation for beam loading can be achieved mainly by a phase shift combined with only a small increase of the input power. The bulk of the beam power stems from a decrease of the power that is lost in the terminating load.

### Table I

<table>
<thead>
<tr>
<th>R.f. parameters of the 300 GeV Synchrotron</th>
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<tbody>
<tr>
<td>Peak r.f. voltage per revolution ($\phi_s = 45^\circ$)</td>
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<tr>
<td>Accelerating frequency</td>
</tr>
<tr>
<td>Frequency swing</td>
</tr>
<tr>
<td>Number of accelerating stations</td>
</tr>
<tr>
<td>Number of accelerating guides and power amplifiers per station</td>
</tr>
<tr>
<td>Length of accelerating station</td>
</tr>
<tr>
<td>$R/Q$ per unit length</td>
</tr>
<tr>
<td>Total power required without beam</td>
</tr>
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It is very fortunate that this condition prevails for more than 90% of the accelerating cycle.

During the time the synchrotron is filled from its booster injector and during the subsequent period of reduced rate of acceleration — the so-called front porch — no increase in input power is required to cover beam loading. There remains, however, a short period, about 30 ms, between the end of the front porch and the transition energy, when the average beam-induced field and the desired accelerating field nearly oppose each other and a substantial increase of input power is required to compensate for beam loading. With the parameters considered and for $3 \times 10^9$ particles per pulse the required increase is about 50%.

**IV. ACCELERATING STRUCTURES**

It follows from eq. [2] that the accelerating structure should have as high a value of $R/Q$ as possible while its losses are not of great importance. The optimum group velocity for the desired guide length and given frequency swing is 0.062 c but the optimum is not critical.

A type of structure that appears to be particularly promising for the given purpose consists of a circular or rectangular guide loaded with transverse bars. Structures of this type have been studied in a number of laboratories for linear accelerator applications. However, these structures have in general been optimized for shunt impedance $R$ instead of $R/Q$. We have, therefore, started an experimental optimizing study of bar loaded structures and first results are reported below.

In all our models the loading bars of adjacent cells have been made parallel since this leads to partial cancellation of magnetic and capacitive coupling which results in group velocities of about the right magnitude. All structures tested are of the backward wave type.

Models of structures with and without drift tubes have been tested.

A cross-section of a model without drift tubes is shown in Fig. 1 (solid lines). The measurements were done at a frequency of about 2860 MHz and the dimensions shown refer to that frequency. The real dimensions at 180 MHz would be larger by a factor 16.

Values of $R/Q$ and group velocity have been measured with three different cell lengths, corresponding to the $\pi/2$, $\pi/3$ and $\pi/4$ modes at $v = c$, and results are given in Table II. The $R/Q$ values given are scaled to 180 MHz and refer to the fundamental space harmonic.

As can be seen from Table II, $R/Q$ improves with decreasing cell length but the group velocity tends to become too high.

Fortunately, since one is near to a cancellation of magnetic and electric coupling, relatively large changes of group velocity can be achieved by fairly small perturbations of the field distribution, such that $R/Q$ is not greatly affected. Thus

![Fig. 1 - Bar-loaded structure without drift tubes.](image)

![Fig. 2 - Bar-loaded structure with drift tubes.](image)

**TABLE II**

<table>
<thead>
<tr>
<th>Mode</th>
<th>$R/Q$ at 180 MHz $\Omega$/h</th>
<th>$v_g/c$</th>
</tr>
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<tbody>
<tr>
<td>$\pi/2$</td>
<td>350</td>
<td>0.063</td>
</tr>
<tr>
<td>$\pi/3$</td>
<td>470</td>
<td>0.080</td>
</tr>
<tr>
<td>$\pi/4$</td>
<td>530</td>
<td>0.088</td>
</tr>
</tbody>
</table>
it was found that the group velocity can be conveniently reduced by means of small circular pedestals which are inserted between the ends of the loading bars and the guide wall as shown in dotted lines in Fig. 1.

Since the pedestals are situated in a region of high magnetic field their insertion leads to an increase of the frequencies of all modes. However, the frequency shift is larger near the π-mode where the field is concentrated around the loading bars. Hence, the mode spacing is reduced, i.e., the group velocity is lowered. Results which have been obtained with a three-cell model suggest that it should be easy to obtain a group velocity of 0.062 c at π/3 mode and v_p = c without altering the R/Q.

The addition of drift tubes brings about an increase in R/Q but also an increase of capacitive coupling from cell to cell, making it more difficult to obtain sufficiently low values of group velocity. A series of drift-tube loaded structures of the geometry and dimensions shown in Fig. 2 have been tested. The nominal frequency for these models was 1300 MHz and the cell length was such as to yield v_p = c at π/2 mode.

The variable parameter was the drift-tube length and Table III shows results for 5 different ratios, g, of drift-tube length to cell length. In each case, the distance between faces of pedestals (a in Fig. 2) was adjusted to yield the right resonance frequency (except for g = 0.39 where the frequency was 2% too low, corresponding to v_p = 0.98 c).

It may be concluded that R/Q values approaching 600 Ω/m at 180 MHz are obtainable together with group velocities which, although somewhat higher than ideal for the given parameters, are within the limits of 90% performance. The optimum seems to occur at about g = 0.35 were the net figure of R/Q to be inserted into eq. [2] is 560 Ω/m. It is interesting to compare this figure with that of the conventional iris-loaded guide which, at 180 MHz, would only be about 250 Ω/m.

### Table III

<table>
<thead>
<tr>
<th>Drift tube length</th>
<th>R/Q at 180 MHz</th>
<th>v_p/c</th>
</tr>
</thead>
<tbody>
<tr>
<td>over cell length, g</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.390</td>
<td>600</td>
<td>0.088</td>
</tr>
<tr>
<td>0.347</td>
<td>600</td>
<td>0.081</td>
</tr>
<tr>
<td>0.294</td>
<td>570</td>
<td>0.077</td>
</tr>
<tr>
<td>0.243</td>
<td>550</td>
<td>0.073</td>
</tr>
<tr>
<td>0.191</td>
<td>500</td>
<td>0.067</td>
</tr>
</tbody>
</table>

**REFERENCES**

(2) W. Schnell: CERN Report 64-10 (1964).

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**CASCADE SYNCHROTRONS FOR ENERGIES UP TO 100 GeV**

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(Presented by Y. Kobayashi)

Cascade synchrotron which consists of a main synchrotron ring, a booster synchrotron ring and a pre-injector was first suggested by R.R. Wilson (1) and the usefulness of the scheme for super high energy accelerators was independently pointed out by M. Sands (2). Thenceforce, the cascade synchrotron was extensively studied at several places as a hopeful device to attain a super high energy. Here, we have a question what would be the practical lower limit of energy of the cascade scheme. Our interest was especially on the feasibility and the econo-