So you are making money in financial markets. Should you tell your friends how?

Damien Challet  
*Institute for Scientific Interchange, Viale S. Severo 65, 10133 Torino, Italy*  
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Constant price impact functions widely used in financial modelling strongly advise in favour of letting trusted friends exploit one’s own arbitrage opportunities because they prevent correct arbitrage removal. Starting from the example of chain arbitrage exploitation, a consistency criterion is proposed. It is only fulfilled by non-constant impact functions. The role of the feedback of sequential market orders of the same kind on the order book is crucial for ensuring consistency at the smallest time scale.

I. INTRODUCTION

There are several starting points for studying financial market dynamics theoretically. The most widely used is to assume that the actions of all traders and external news result collectively into so large fluctuations that any predictability pattern is neither detectable nor exploitable. This view, dating back at least to Bachelier [1], is the basis of the Efficient Market Hypothesis (EMH) [2] and is very convenient for elaborating option pricing theories [3]. Behavioural finance starts from EMH and incorporates deviations from rational expectations in the behaviour of the agents in an attempt to explain anomalous properties of financial markets (see [4] for a recent review). Another approach is to inject a controllable amount of predictability and study how the traders exploit and remove it in order to understand what the dynamical road to efficiency is [5, 6].

Even the champions of EMH agree that there are temporary anomalies, due for instance to uninformed traders (sometimes called noise traders), that arbitrageurs tend to cancel. Thus the random walk hypothesis must be viewed as an extreme assumption describing an average idealised behaviour that does not describe every detail of the microscopic price dynamics. And indeed extreme assumptions are most useful in any theoretical framework. This is why the opposite assumption is worth considering: the discussion will start by focusing on a single transaction and investigates how to exploit perfect knowledge about it. Trader 0 is active at time \( t \); he buys/sells a given amount of shares \( n_0 \), leading to (log-)price change \( r(t) = r_0 \), where \( t \) is in transaction time, \( t \) being the \( t \)-th transaction. Even more, one also assumes that trader 1 has perfect information about \( t \) and \( r_0 \). What the latter must do in order to exploit his piece of information is clear.

A related discussion is found in Ref. [7] which discusses (among other topics) the exploitation of an isolated pattern; its main result is that the pattern does not disappear; on the contrary, its very exploitation spreads it around \( t \). Therefore, this work raises the question of how microscopic arbitrage removal is possible at all. In addition, since the arbitrage is still present, it is still exploitable.

This paper aims at answering these two questions. It shows first the need to change the way price impact and speculation is incorporated in financial literature. Indeed its immense majority relies on two fundamental assumptions: first constant and often symmetric price impact functions. The second point is how arbitrage is removed. Current literature often restricts its attention to single-time price returns, without taking into account the fact that speculation is inherently inter-temporal. This is simpler but wrong. Indeed, one does not make money by the transactions themselves, but by holding a position, that is, by doing nothing.

Both assumptions are useful simplifications that made possible some understanding of the dynamics of market models. However, since the discussion on arbitrage in market models is generally in discrete time, one could in principle argue that one time step is long enough to include holding periods, but this is inconsistent with the nature of most financial markets. Indeed, the buy/sell orders arrive usually asynchronously in continuous time, which rules out the possibility of synchronous trading; at a larger time scale, the presence of widely different time scales also rules out perfect synchronicity.

Once arbitrage exploitation is considered at its most minute temporal level, that is, when its ineluctable inter-temporality is respected, a simple consistency criterion for price impact functions emerges. If violated, a paradoxical arbitrage chain exploitation is possible, or equivalently, the gain of a money-making trader is not decreased if he informs his trusted friends of the existence of arbitrage opportunities. As we shall see, the dynamics of price impact functions is a key element of their consistency.

By focusing on the exploitation a single transaction isolated in time, and by assuming that trader 1 experiences no difficulty in injecting his transactions just before and after \( t \), his risk profiles with respect to price fluctuations and uncertain position holding period are irrelevant and will be neglected.
II. THE PARADOX

The price impact function $I(n)$ is by definition the relative price change caused by a transaction of $n$ (integer) shares ($n > 0$ for buying, $n < 0$ for selling); mathematically,

$$p(t + 1) = p(t) + I(n),$$

where $p(t)$ is the log-price and $t$ is in transaction time. The above notation misleadingly suggests that $I$ does not depend on time. In reality, not only $I$ is subject to random fluctuations (which will be neglected here), but also, for instance, to strong feed-back from the type of market orders which has a long memory (see e.g. [8, 9, 10] for discussions about the dynamical nature of market impact). Neglecting the dynamics of $I$ requires us to consider specific shapes for $I$ that enforce some properties of price impact for each transaction, whereas in reality they only hold on average. For example, one should restrict oneself to the class of functions that makes it impossible to hold on average. For example, one should restrict oneself to the class of functions that makes it impossible to predict gain for each transaction, whereas in reality they only hold on average.

The most intuitive (but wrong) view of market inefficiency is to regard price predictability as a scalar deviation from the unpredictable case: if there were a relative gain of $g_1$ share for transaction of $n$ shares irrespective of the order at which time $t$, according to this view, one should exchange $n$ shares so as to cancel perfectly this anomaly, where $n_1$ is such that $I(n_1) = 0$. This view amounts to regarding predictability as something that can be remedied with a single trade. However, the people that would try and cancel $n_0$ would not gain anything by doing it unless they are market makers who try to stabilise the price. The speculators on the other hand make money by opening, holding, and closing positions. Hence one needs to understand the mechanisms of arbitrage removal by the speculators.

Trader 1, a perfectly (and possibly illegally) informed speculator, will take advantage of his knowledge by opening a position at time $t - 1$ and closing it at time $t + 1$. It is important to be aware that if one places an order at time $t$, the transaction takes place at price $p(t + 1)$. Provided that trader 0 buys/sells $n_0$ shares irrespective of the price that he obtains, the round-trip of trader 1 yields a monetary gain of

$$g_1 = n_1 e^{I(t+2)} - e^{I(t)} = n_1 e^{p_0} [e^{I(t+2)} - e^{I(n_1)}],$$

and a relative gain of

$$\hat{g}_1 = \frac{n_1 [e^{p(t+2)} - e^{p(t)}]}{n_1 e^{p(t)}} = e^{I(t+2)} - e^{I(n_1)} - 1,$$

where $p_0$ is the log-price before any trader considered here makes a transaction. Since $I(n)$ generally increases with $n$, there is an optimal $n^*_1$ number of shares that maximises $g_1$. The discussion so far is a simplification, in real-money instead of log-money space, of the one found in Ref. [3]. One should note that far from diminishing price predictability, the intervention of trader 1 increases the fluctuations. Therefore, in the framework of constant price impact functions, an isolated arbitrage opportunity never vanishes but becomes less and less exploitable because of the fluctuations, thus the reduction, of signal-to-noise ratio caused by the speculators.

It seems that trader 1 cannot achieve a better gain than by holding $n^*_1$ shares at time $t$. Since the actions of trader 1 do not modify in any way the arbitrage opportunity between $t - 2$ and $t + 2$, he can inform a fully trusted friend, trader 2, of the gain opportunity on the condition that the latter opens his position before $t - 1$ and closes it after $t + 1$ so as to avoid modifying the relative gain of trader 1. For instance, trader 2 informs trader 1 when he has opened his position and trader 1 tells trader 2 when he has closed his position. From the point of view of trader 2, this is very reasonable because the resulting action of trader 1 is to leave the arbitrage opportunity unchanged to $r_0$ since $p(t + 1) - p(t - 1) = r_0$. Trader 2 will consequently buy $n^*_2 = n^*_1$ shares at time $t - 2$ and sell them at time $t + 2$, earning the same return as trader 1. This can go on until trader 1 has no fully trusted friend. Note that the advantage of trader 1 is that he holds a position over a smaller time interval, thereby increasing his return rate; in addition, since trader 2 increases the opening price of trader 1, which results into a prefactor $e^{I(n_2)}$ in Eq. (1), the absolute monetary gain of trader 1 actually increases provided that he has enough capital to invest. Before explaining why this situation is paradoxical, it makes sense to emphasise that the gains of traders $i > 0$ are of course obtained at the expense of trader 0, and that the result the particular order of the traders’ actions is to create a bubble which peaks at time $t + 1$.

The paradox is the following: if trader 1 is alone, the best return that can be extracted from his perfect knowledge is $\hat{g}_1(n^*_1)$ according to the above reasoning. When there are $N$ traders in the ring of trust, the total return extracted is $N$ times the optimal gain of a single trader. Now, assume that trader 1 has two brokering accounts; he can use each of his accounts, respecting the order in which to open and close his positions, effectively earning the optimal return on each of his accounts. The paradox is that his actions would be completely equivalent to investing $n^*_1$ and then $n^*_1$ from the same account. In particular, in the case of $I(n) = n$, this seems a priori exactly similar to grouping the two transactions into $2n^*_1$, but this results of course in a return smaller than the optimal return for a doubled investment. Hence, in this framework, trader 1 can earn as much as pleases provided that he splits his investment into sub-parts of $n^*_1$ shares whatever $I$ is, as long as it is constant.

Two criticisms can be raised. First, the ring of trust
must be perfect for this scheme to work, otherwise a Prisoner’s dilemma arises, as it is advantageous for trader \(i+1\) to defect and close his position before trader \(i\). In that case, the expected return for each trader is of order \(1/N\), as in Ref [2].

But more importantly, one may expect that the above discussion relies crucially on the fact that \(n_0\) does not depend on the actual price, or equivalently that trader 0 wishes to buy or to sell a predetermined number of shares. As we shall see in the second part of this paper, the paradox still exists even if trader 0 has a fixed budget \(C\) (which is more likely to arise if trader 0 intends to buy).

But this paradox seems too good to be present in real markets. As a consequence, one should rather consider its impossibility as an a contrario consistency criterion for price impact functions. The final part of this paper explores the dynamics of price impact functions.

III. FINITE CAPITAL

When trader 0 has a finite capital, the number of shares that he can buy decreases when traders 1, 2, \ldots increase the share price before his transaction. Let us assume that trader 0 has capital \(C\) that buys \(n_0\) shares at price \(e^{p_0}\). The price he obtains is different from \(e^{p_0}\) because of his impact: the real quantity of shares that \(C\) can buy is therefore self-consistently determined by

\[
n_0^{(0)} = \frac{n_0}{e^{I(n_0^{(0)})}}.
\]

We shall first focus on the case where the self impact is neglected, or equivalently where trader 0 has a restricted budget with flexible constraint since it is leads to simpler mathematical expressions.

When only trader 1 is active before trader 0, \(n_0\) becomes \(n_0^{(1)} = n_0/e^{I(n_1)}\), thus the gain of trader 1 is

\[
g_1/e^{p_0} = n_1[e^{I(n_0/e^{I(n_1)})} - e^{I(n_1)}].
\]

It is easy to convince oneself that there is always an \(n_1^*\) provided that \(n_0\) is large enough. Trader 1 must now be careful when communicating the existence of the arbitrage to trader 2, since the latter decreases the price return caused by trader 0. Indeed, assuming that trader 1 invests \(n_1^*\) whatever trader 2 does, his gain is given by

\[
g_{1,2}(n_1^*, n_2)/e^{p_0} = n_1^*[e^{I(n_0/e^{I(n_1^*)})+I(n_2)} - e^{I(n_2)}] - n_1[e^{I(n_0/e^{I(n_1^*)})+I(n_2)} - e^{I(n_2)}].
\]

He would therefore lose \(\Delta g_1(n_2) = g_1^* - g_{1,2}(n_1^*, n_2)\), which must be compensated for by trader 2. Without compensation payment, the gain of the latter is

\[
g_{2}/e^{p_0} = n_2[e^{I(n_0/e^{I(n_1^*)})+I(n_2)} - e^{I(n_2)}].
\]

The case where trader 0 has a strict budget constraint is obtained by replacing \(n_0\) by \(n_0^{(0)}\) in Eqs (5), (6) and (7).

The paradox exists if trader 2 has a positive total gain, i.e., \(G_2 = g_2 - \Delta g_1 > 0\). In order to investigate when this is the case, one must resort to particular examples of price impact functions.

Empirical research showed that \(I\) is a non-linear, concave function \([11, 12, 13, 14, 15]\). Although there is no consensus on its shape, logarithms and power-laws are possible candidates. From a mathematical point of view, logarithmic functions allow for explicit computations, while power-laws must be investigated numerically.

A. Log price impact functions

The generic impact function that will be studied is

\[
I(|x|) = \gamma \log(|\lambda x|); \quad \lambda > 1,
\]

where \(\gamma < 1\) is related to the liquidity and brings down the impact to reasonable levels: if \(\lambda x = 100\) and \(\gamma = 0.01\), the price is increased by about 5%.

For the sake of comparison, we address the case of infinite capital: by differentiating Eq (4) with respect to \(n_1\), the optimal number of shares to invest is

\[
n_1^* = \frac{n_0}{(\gamma + 1)^{1/\gamma}},
\]

which simplifies to \(n_0/2\) if \(\gamma = 1\). The optimal monetary gain is given by

\[
g_1^* = e^{p_0}n_0\frac{\gamma}{(\gamma + 1)^{1+1/\gamma}}.
\]

In the case of finite capital, the optimal number of shares to invest for trader 1 is

\[
n_1^* = [n_0(1 - \gamma)]^{1/(1 - \gamma)};
\]

The optimal gain is given by

\[
g_1^* = e^{p_0}n_0\frac{\gamma}{(1 - \gamma)^{1-1/\gamma}} = C\frac{\gamma}{(1 - \gamma)^{1-1/\gamma}},
\]

which is linear in \(n_0\), in contrast to its super-linearity in Eq (4): the limited budget of trader 0 helps to remove predictability.

The gain of trader 1 in the presence of trader 2 is

\[
g_{1,2}(n_1^*, n_2) = e^{p_0}n_0(1 - \gamma)^{1/\gamma}\left(\frac{1}{(1 - \gamma)n_2^2} - 1\right);
\]

he therefore would lose

\[
\Delta g_1(n_2) = g_1^* - g_{1,2}(n_1^*, n_2) = e^{p_0}n_0(1 - \gamma)^{1/\gamma-1}\left[1 - \frac{1}{n_2^2}\right].
\]
Trader 2 optimises

\[ \frac{G_2}{e^{p_0}} = [g_2 - \Delta g_1] e^{-p_0} = \left[ \frac{n_0}{1 - \gamma} \right]^{\gamma \beta} n_2^{1 - \gamma} - n_2^{1 + \gamma} - \Delta g_1(n_2) \]

(14)

with respect to \( n_2 \). The paradox survives in the regions of the parameter space such that \( G_2 > 0^* \) and \( n_2^* > 1 \). Numerical investigations show that \( G_2^* > 0 \) is always true. From Fig. 1 one sees that the paradox exists provided that scaled \( n_0 \) is large enough (\( \lambda n_0 > e^{1 + \gamma/2} \) for \( \gamma \ll 1 \)) and \( n_0 \) small enough, which is compatible with realistic values. Note that \( g_1^* \) and \( G_2^* \) are increasing functions of \( \gamma \), since this parameter tunes the price return caused by \( n_0^{(1)} \) and \( n_0^{(2)} \).

If trader 0 has a strict budget, the number of shares he can afford is determined self-consistently by \( n_0^{(0)} = C/e^{p_0 + \gamma \log n_0^{(0)}} \) without the intervention of the trader 1, that is, \( n_0^{(0)} = n_0^{1/(\gamma + 1)} \); after trader 1 has opened his position, trader 0 can only afford

\[ n_0^{(1)} = n_0^{(0)} / n_1^{\gamma/(\gamma + 1)} = \left[ \frac{n_0}{n_1} \right]^{\frac{\gamma}{\gamma + 1}} \]

(15)

shares. Similarly,

\[ n_0^* = n_0^{1/(\gamma + 1)} \left[ 1 - \gamma^2/(\gamma + 1) \right]^{\frac{\gamma + 1}{\gamma + 1}}, \]

(16)

hence the number of shares to invest is lowered further in comparison with Eq (10). All the expressions of the respective gains of trader 1 and 2 do not simplify to neat and short expressions and no analytical solution to the maximisation of \( G_2 \) can be found. Numerical maximisation yields the dashed boundary line in Fig. 1. The strict constraint increases the minimum \( n_0 \) needed for making the paradox possible when \( \gamma \) is small enough. At \( \gamma \simeq 0.66 \), the boundary lines cross; this may be due to the fact that trader 2 must pay a smaller compensation to trader 1 when \( \gamma \) increases.

**B. Power-law impact functions**

Several papers suggest a power-law price impact function \( I(|x|) = |x/\lambda|^\beta \) with \( \lambda \geq 1 \) and \( \beta \in [0.4, 0.8] \) [13, 15, 16]. With this choice of functions, it is not possible to maximise explicitly \( g_1 \) and \( G_2 \).

Performing all the maximisations of the finite capital case numerically, one finds that the picture of the log impact function is still valid in this case (see Fig. 2), in particular for \( I(x) \propto x \). Interestingly, this is the most common choice in the literature on agent-based models, option pricing, etc. It was derived analytically by Kyle [17] under the assumption of linearity between private information and insiders’ order flow. More recently, Farmer [18] uses it as simple example of a function that prevents making trading profits from a round trip. And certainly, it may seem the least arbitrary, since it does not seem to impose any particular choice of \( \beta \). Hence constant \( I(x) \propto x \) is to be banished. But power-laws are inconsistent for all possible real-market values of \( \beta \); once again, this shows that constant price impact functions are to be avoided.

**IV. FEEDBACK**

The only sure way out from inconsistent price impact functions is to take into account the dynamics of the order book, particularly the reaction of the order book...
to a sequence of market orders of the same kind. There is a consensus that the impact of a second order is smaller than that of the first one of the same kind and size \( \xi \); mathematically, \( I(n,t) + I(n,t+1) < 2I(n,t) \).

For example, a recent dynamical theory of market impact \( \xi \) states that the impact of a trader is not only instantaneous, but is a decreasing function \( G_0(\Delta t) = \Gamma_0/(\eta^2 + \Delta t^2)^{3/2} \) where \( \Delta t \) is the time (in number of transactions) since the transaction. To be more precise, the impact due to a single trade of volume \( n \Delta t \) time steps after the transaction is \( \text{sgn}(n) \ln|n|G_0(\Delta t) + \xi(\Delta t) \), where \( \xi(\Delta t) \) is a white noise term with variance proportional to \( \Delta t \) and \( G_0(x) \) is a monotonically decreasing function. Therefore, the average total impact of two consecutive trades of the same kind volume is \( \ln n[G_0(0) + G_0(1)] < 2\ln nG_0(0) \) (the reader is referred to Ref. \( \xi \) for more details); the important point is that \( G_0 \) is slowly decreasing, almost constant for small arguments, suggesting a value of relative impact of two consecutive trades of the same type, denoted by \( \kappa \) which depends on the stock considered, but always very close to 1.

Therefore, for our purpose, we shall assume that a second market order of the same size and type has an impact described by \( I_2(n) = \kappa I(n) \) with \( 0 < \kappa < 1 \), the third \( I_3(n) = \kappa^2 I(n) \), etc., and that consecutive market orders on one side do not influence the price impact function of the other side.

This assumption is also supported by an empirical work on the long memory of market order signs \( \xi \). Whatever the precise shape of price impact, Fig. 12 of this work is strikingly compatible with the assumption that the whole price impact function is multiplied by a given constant after each successive market order of the same kind, hence, can be used to measure \( \kappa \); this figure plots the average cumulated price impact of two market orders of the same type, with volume \( n_1 \) and \( n_2 \) respectively, conditional on volume \( n_1 \), that is

\[
I(n_1) + \int P(n_2|n_1)I(n_2|n_1)dn_2,
\]

where \( P(n_2|n_1) \) is the probability that the size of the second market order is \( n_2 \) given the fact that the size of the first one was \( n_1 \). The important difference from our case is that the two orders need not be placed immediately one after the other; therefore, the value of \( \kappa \) that can be estimated from this figure is a lower bound with respect to the ideal situation considered here. Assuming that most of the consecutive market orders of the same kind come from a large order that has been split, which is believed to be the cause of the long memory of the market order signs \( \xi \), it also makes sense to suppose that their sizes are comparable; pushing this line of thought to its extreme, one approximates \( P(n_2|n_1) \approx \delta(n_2 - n_1) \), and obtains finally from the aforementioned figure

\[
I(n_2|n_1) \approx I_\kappa(n_2) \approx (0.14 \pm 0.04)I(n_2),
\]

that is, \( \kappa = 0.14 \pm 0.04 \). Similarly, one checks that a third order also multiplies \( I_\kappa \) by \( \kappa = 0.09 \pm 0.02 \). Since the values of these averages are not statistically different, we shall assume that \( \kappa \approx 0.12 \). We note that \( \kappa \) seems to decrease for each subsequent market order of the same kind in Fig. 12 of Ref. \( \xi \).

When writing the gains of trader 1 and 2, one must be very careful with the order of the transactions. Starting with the case of infinite capital of trader 0, the gain of trader 1 is

\[
g_1 = n_1[\epsilon(I(n_1) + I_\kappa(n_0) - I(n_1) - I_\kappa(n_1))],
\]

which makes it clear that there is still a non-zero optimal number of shares \( n_1^* \) to invest. Similarly, the gain \( g_{1,2} \) is now

\[
g_{1,2} = n_1^*[\epsilon(I(n_2) + I_\kappa(n_1) + I_\kappa(n_0) - I(n_1) - I_\kappa(n_1))],
\]

while that of trader 2, without his payment to trader 1 is

\[
g_2 = n_2(\epsilon I(n_2) + I_\kappa(n_1) + I_\kappa(n_0) - I(n_1) - I_\kappa(n_1)).
\]

In the case of log price impact functions, the optimal number of shares and gain of trader 1 are

\[
n_1^* = \frac{n_0^\gamma}{(\gamma + 1)^{1/\gamma}},
\]

and

\[
g_1^* = e^{\gamma_0}n_0^\gamma(\gamma + 1)^{1/\gamma}/(\gamma + 1)^{1/\gamma},
\]

These two equations already show that the reaction of the limit order book reduces the gain opportunity of player 1. Adding trader 2 will reduce further the impact of trader 0, hence the gain of trader 1, and, as before, trader 2 should pay for it. In this case, the reduction of gain of trader 1 is

\[
\Delta g_1/e^{\gamma_0} = [g_1^* - g_1(n_1^*, n_2)]e^{-\gamma_0} = n_0^\gamma(\gamma + 1)^{1/\gamma}/(\gamma + 1)^{1/\gamma},
\]

where the gain that trader 2 optimises is

\[
G_2/e^{\gamma_0} = n_2^\gamma\left[\frac{1}{n_2^\gamma} - 1\right] - \Delta g_1/e^{\gamma_0}.
\]

Trader 1’s impact functions are \( I_\kappa \) when he opens his position and \( I \) when he closes it, which is an additional cause of loss for trader 1, which must be also compensated for by trader 2. Fortunately for the latter, his impact functions are \( I \) when opening and \( I_\kappa \) when closing his position. Therefore, provided that \( \kappa \) is large enough so as not to make \( I_\kappa(n_0) \) too small, trader 2 can earn more than trader 1 in some circumstances.

As above, impact functions are inconsistent when \( G_2^* > 0 \), \( n_1^* > 1 \) and \( n_2^* > 1 \) for log impact functions. It turns
The case of power-law impact functions is different, since $G_2^* > 0$ and $n_1^* > 0$ are sufficient conditions for inconsistent impact functions. When $\kappa$, is small enough, $G_2^* > 0$ becomes impossible (Fig. 3).

The values of critical $\kappa$ stand between the ones reported by the empirical studies previously mentioned [8, 9]. The values of $\kappa$ they report or imply are completely different from each other, and so are their fundamental assumptions. However our assumption that the feedback consists in multiplying the whole impact function by a constant is compatible with the two of them. Regarding the dynamical theory of Ref. [8], we note that assuming that the reaction of the order book to limit order markets is not a purely white noise, but anticorrelated to the sign of the market orders would decrease the effective value of $\kappa$; therefore, this work gives an upper bound for $\kappa$ in our case. On the other hand, Ref. [9] reports the total price impact of two consecutive market orders irrespective of the time that separates them; this is not equivalent to our assumption of market orders in quick succession. Therefore, the value of $\kappa$ given by this work is a lower bound for the case presented here. We argue therefore that in the situation considered in this paper, real-market $\kappa$ is probably close to its critical value.

We emphasise that it is the feedback that, when large enough, makes impact functions consistent. Current literature on limit order price impact functions does make it possible to be conclusive about the consistency of real markets price impact functions.

V. CONCLUSIONS

Financial markets may ensure consistent market price impact functions at the most microscopic dynamical level by the dynamics of the impact function. The paradox proposed in this paper provides a simple and necessary condition of consistency for price impact functions, and shows the need to stop approximating them with constant linear functions in financial literature. As soon as a strong enough feedback is present, the paradox does not exist, even with linear price impact functions. As a consequence, one may still use such functions for discussing market efficiency, but they should not be constant.

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[18] If trader 2 were not a good friend, trader 1 could in principle ask trader 2 to open his position after him and to close it after him, thus earning more. But relationships with real friends are supposed to egalitarian in this paper.
[19] Only the points corresponding to the three largest $n_1$ have been used to estimate $\kappa$. 